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Journal of the
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EFFECTIVE LENGTH OF FRAMED COLUMNS

By Thomas C. Kavanagh,[1] F. ASCE

FOREWORD

The papers that form the ASCE-CRC Symposium on Metal Compression Members are representative of all phases of Column Research Council activities. Several papers represent summaries of both past and current research, prepared specifically to assist in the preparation of the "Guide to Design Criteria for Metal Compression Members." Also included are reports on recently completed research on pony truss bridges and on plate girders.

In the first paper, (January, 1960), B. G. Johnston documented the inception of the CRC and the many activities undertaken by the Council.

SYNOPSIS

The assumption in many structural specifications that the effective length of columns is equal to the actual length may be extremely conservative in some cases and very hazardous in others. Framed columns may have effective lengths even approaching an infinite multiple of the actual length, depending on the end restraints.

This paper reviews the current state of knowledge relating to effective lengths, leading from the rigorous, though laborious, procedures to the development of rational approximations using formulae, charts and nomograms, which, together with a proper classification of type of action, permit a rapid selection of the proper effective length for design.

Note.—Discussion open until June 1, 1960. Separate Discussions should be submitted for the individual papers in this symposium. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Structural Division, Proceedings of the American Society of Civil Engineers, Vol. 86, No. ST 2, February, 1960.

[1] Parnter, Praeger-Kavanagh, Engrs. New York, N. Y.

INTRODUCTION

In order to design a column, a structural engineer must know its "effective length," or, what is equivalent thereto, the type and magnitude of end conditions or restraints on the column. Elementary theory teaches that pin-ended column has an effective length equal to its actual length, but, theoretically at least, it is possible for the effective length of columns to vary anywhere up to an infinite multiple of the actual column length.

The Column Research Council has undertaken in the past 15 years to gather and consolidate the work of many investigators on this subject including, besides the writer[1][2][3][4][5][6][7],[2] Friedrich Bleich[8][9], H. Bleich, N. J. Hoff[10][11][12], E. E. Lundquist[13][19], George Winter[14], P. P. Bijlaard[15], Nathan M. Newmark[20], A. Slavin[16], J. G. Perri[17], O. G. Julian[18] and others to whom full acknowledgment is made. The results, in the form of satisfactory and simple approximations, furnish a rational basis for inclusion in specifications of provisions for the determination of the proper effective length of framed columns, and will represent a marked improvement over existing codes.

FUNDAMENTAL CONSIDERATIONS

Compression Member with End Restraints.—The behavior of a compression member, which in practice is almost always a part of a structural framework, is determined by the several restraints imposed by the framework upon the ends of that member. It is perhaps fortunate that one is thus able to focus attention in designing the member upon the individual compression member itself with its so-called "end conditions" imposed by the framework, since it is characteristic of normal design procedure to concentrate on the design of individual members rather than the total framework itself. Even where analysis, rather than design, is involved (as in rating a given column in a bridge for permissible load) it is more convenient to deal with the individual column and its restraints than to consider the framework as a whole.

Rigorous methods of calculating the stability of compression members which are part of rigid jointed frameworks are available and have been checked by tests (1) through (18). These methods involve the application of any of the classical or modern techniques of indeterminate analysis, modified to take account of the effect of axial loads and plastic action on the stiffness or rigidity of the member. These procedures are far too complex for, and are not directly applicable to, routine design except through the equivalence of these procedures to the determination or approximation of end restraints on the component members of the framework.

End Restraints.—These end restraints are of various types, may occur in several directions and may also vary in magnitude as the axial loads in the framework change. The restraints themselves may be flexural or torsional restraints, comprising a rotational restraint or rigidity offered by the framework against the angular rotation of the ends of the compression member in either or all of the three normal planes at each end. Or, the restraints may be translational, sometimes denoted "directional," involving resistance by the framework against linear movement ("deflection" or "translation") of the

[2] Numerals in parentheses, thus (1), refer to corresponding items in the Bibliography.

ends of the compression member in three normal directions at each end. The numerical magnitude of the restraints, or the corresponding "spring constants" (measured in typical units as foot-pounds per radian, or as pounds per inch respectively), need not be (and actually are not) constants, but vary with loads. The variation is further complicated by non-linear stress-strain relations in the plastic range.

Engineers familiar with moment distribution will recognize that a flexural restraint against rotation of one end of a compression member offered by an adjacent member in the same plane is the same as the stiffness of the adjacent member. The absolute value of this in the elastic range (where no axial load exists in the restraining member) is known to be:

$$K = \frac{4\,E\,I}{L} \qquad \text{(1a)}$$

for the far end fixed, and

$$K'' = \frac{3\,E\,I}{L} \qquad \text{(1b)}$$

for the far end pinned (Fig. 1).

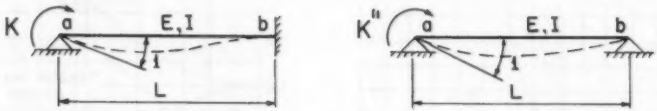


FIG. 1

To modify for plastic action and the effects of axial load P (tension or compression) in the restraining member, these values (1) become, respectively:

$$K : 4\,S\,\frac{E_t I}{L} \qquad \text{(2a)}$$

and

$$K'' : 4\,S''\,\frac{E_t I}{L} \qquad \text{(2b)}$$

where E_t is the tangent modulus of elasticity, and S and S'' are stiffness factors taken from Fig. 2[26] or from extended tabular values[19].

Engineers working with moment distribution on rectangular frames are also familiar with the concept that a translational restraint offered by a member against horizontal movement of its end (when no axial leads are present) has either of the following values of limiting translational stiffness of that member (Fig. 3):

$$T : 12\,\frac{EI}{L^3} \qquad \text{(3a)}$$

for the end b fixed, and

$$T'' : 3\,\frac{EI}{L^3} \qquad \text{(3b)}$$

for the end b pinned.

To modify for plastic action and the effects of axial load P (tension or compression) in the member, these values[6] become, respectively:

$$T : \frac{E_t I}{L^3} \left[8S(1+C) - \frac{PL^2}{E_t I} \right] \quad \text{(4a)}$$

and

$$T'' : \frac{E_t I}{L^3} \left[4S'' - \frac{PL^2}{E_t I} \right] \quad \text{(4b)}$$

where P is the absolute value of the load in the member, and S, S'' and C are stiffness and carry-over factors taken from Fig. 2, or from the corresponding published tabular values.[19]

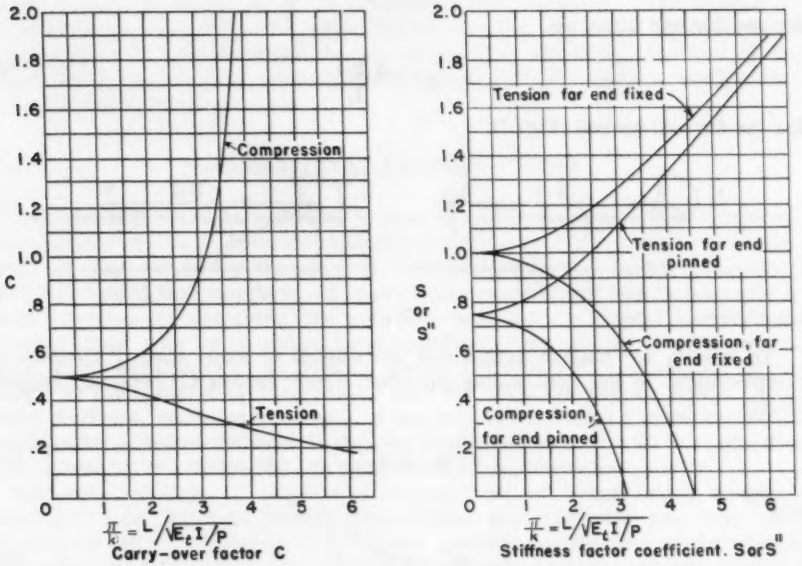


FIG. 2

Simplification of Frameworks.—Although the stability of the compression member has been considered in terms of a member in a general space framework, it is convenient to break down the structure into plane frameworks and to consider the buckling of the member in terms of its buckling within the plane of the main framework of which it is a part (such as a main bridge truss), and again as buckling normal to the main framework (sometimes spoken of as "out of the plane") such as that which takes place in the plane of the horizontal bracing of a bridge truss.

Types of Buckling.—The compression member in a plane framework may be characterized as to the type of buckling failure to which it is subjected, as follows:

a) That in which translational rigidity at the ends of the member is so great that little or no translation takes place. (This type of action usually is found in truss frameworks.)

b) That in which both translation and rotation of the ends of the member are possible. (This type of action is usually found in open rectangular rigid frames.)

c) That in which rotational rigidity at the ends of the member is so great that little or no rotation takes place. (An example is a rectangular rigid frame with extremely heavy or stiff upper horizontal cross member.) This type of buckling is usually grouped with b), leaving two simple general categories: buckling with rotation only; and buckling with rotation and/or translation.

A classification of the type of buckling is extremely important in design of compression members, because the presence of translation (sideway) has a major effect for certain types of frameworks (non-triangular).

Limitations.—The treatment which follows applies to steel, aluminum alloys, and other structural metals. In certain instances, notably with structural steel, important simplifications are possible because of the relatively rapid transition between elastic and plastic action, and these will be cited as they occur.

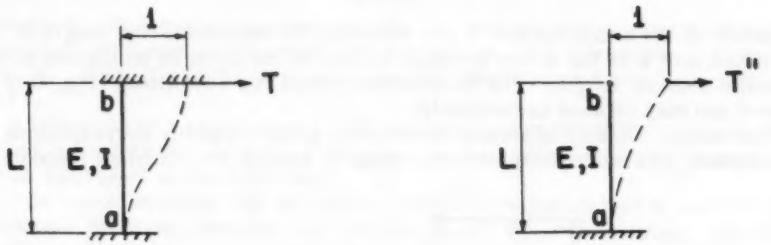


FIG. 3

The writer will confine his remarks to such frameworks where axial-load effects predominate. Engineers familiar with stability problems will recognize in this treatment the solution of an indeterminate type of system by energy methods or other analytic procedures leading to a system of linear simultaneous equations, the vanishing of the determinant of which furnishes the stability criterion; or by convergence methods, such as moment distribution or relaxation, in which stability is tested by a simple physical concept.

The effects of primary or secondary bending moments already present in many frameworks at the instant the system passes from stable to unstable equilibrium are omitted in this discussion. In general these problems differ from the previous in that the concept of stress as a criterion for stability is introduced.

The treatment which follows applies to the buckling of compression members which are straight, homogeneous and of constant cross section.

Effective Length.—The end restraints on a compression member of length L, once calculated or approximated in magnitude, determine the value of the effective length k L to be used in the design column formula. An example of such design formula is the generalized Euler formula:

$$P = \frac{\pi^2 E_t I}{(k L)^2} \quad \text{..... (5a)}$$

$$\sigma = \frac{\pi^2 E_t}{(k\,L/r)^2} \quad \text{..... (5b)}$$

in which P is the critical or buckling load, o represents the critical or buckling average stress, E_t is the effective modulus (tangent modulus), I denotes the moment of inertia of the cross section of the member, r is the radius of

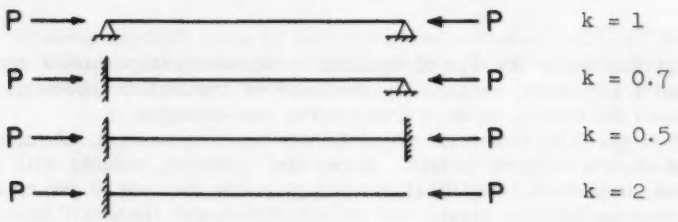


FIG. 4

gyration of the cross section of the member, L represents the length of the member, and k is the factor by which actual length must be multiplied to obtain the reduced length. For certain theoretical end conditions, (Fig. 4) k is shown and may be used immediately.

For actual columns in structures wherein infinite rigidity corresponding to fixed ends, or zero rigidity corresponding to hinges, or complete freedom to

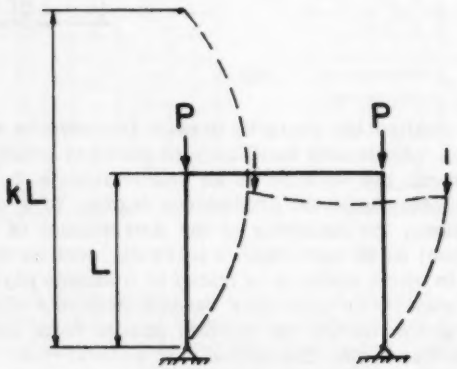


FIG . 5

translate corresponding to a free end, and are not encountered, the k-values may vary between or be greater than any of those shown in Fig. 4. For example, in a rigid frame with hinged bases, as shown in Fig. 5, the value of k may actually approach infinity as the flexural rigidity of the horizontal member decreases toward zero. Thus, in actual structures it is necessary to apply a procedure for determining k more in line with the actual magnitudes and types of restraints existing.

The effective-length factor (k) may be determined from the solution of the appropriate equation[3] for the buckling of a compression member with known end restraints, provided the magnitude of these restraints is known or can be approximated.

a. Buckling of a compression member with known rotational restraints at ends, but with no translation (Fig. 6):

$$(R_a + R_b) + \frac{R_a R_b}{K} + K'' = 0 \quad \ldots\ldots\ldots\ldots\ldots \quad (6)$$

in which R_a and R_b are rotational restraints (foot-lbs per radian) at ends A and B respectively; K is the absolute rotational stiffness of the member, far

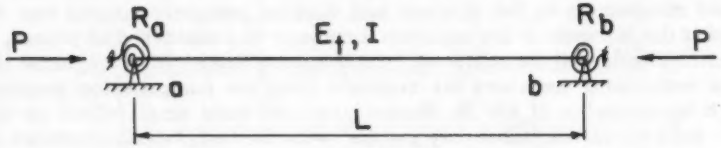


FIG. 6

end fixed. No translation either end (Eq. 2); and K'' represents the absolute rotational stiffness of member, far end pinned. No translation either end (Eq. 2). Both K and K'' are trigonometric functions of the π/k, per Eqs. 2 and 4.

b. Buckling of a compression member with known rotational and translational restraints at both ends. (The translational restraints may be replaced by an equivalent at one end only.)

The same equation (Eq. 6) applies, with the exception that K and K'' are replaced by more complex expressions $\overline{K}$ and $\overline{K}''$, representing rotational stiffnesses of the member with far end fixed and pinned, respectively, but

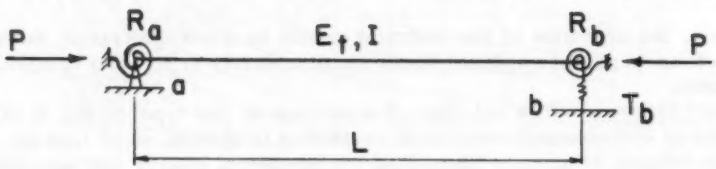


FIG. 7

with the near end translationally restrained by a linear spring of stiffness T_b lbs. per in., as follows (Fig. 7),

in which;

$$(R_a R_b) \frac{R_a R_b}{K} \overline{K}'' = 0 \quad \ldots\ldots\ldots\ldots\ldots \quad (7)$$

$$\overline{K}'' = K \left[\frac{(1-C) + \tau(1+C)}{2} \right] \quad \ldots\ldots\ldots\ldots \quad (8a)$$

$$\overline{K} = K'' \left[\frac{2\tau}{\tau\,(1+C) + (1-C)} \right] \quad \ldots\ldots\ldots\ldots \quad (8b)$$

---

[3] These equations are well known in a variety of forms, all derivable from the fundamental equation of flexure, $\frac{d^2y}{dx^2} = \frac{-M}{E\,I}$.

$$\tau = \frac{T_b - P/L}{T_b + T} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (8c)$$

T_b is the translational restraint at b, T represents the stiffness of a b per Eqs. 4, and C is the carry-over factor from previous charts.

Approximation of End Restraints.—The investigations the writer and others indicate that the end restraints on compression members of frames of the type (a) without translation (such as trusses) can be approximated satisfactorily and without impairing either the safety or economy of the structure by considering each compression member restrained rotationally only by the "adjacent" tension and minor compression members coming into its end. Thus, in a bridge truss, a top chord compression member may be considered restrained rotationally by the diagonal and vertical members coming into its ends, and if the far ends of the adjacent members be considered as pinned, a conservative result will be achieved. Such approximation thus neglects the effects of restraining members far removed from the compression member and which by extension of the St. Venant principle have small effect on the member. Such so-called "three-bay groups" with the compression member in the central bay, form the basis for approximation of end rotational restraints where no translation is involved.

Where two highly stressed compressive members (such as AB and BC, in Fig. 8) are restrained by a common restraining member BE, the restraint of BE should be divided in design between the two compressive members. In

FIG. 8

analysis, the proration of the restraint cannot be made arbitrarily, but would be such as to produce critical loads simultaneously in the two compression members.

Chart Solutions.—The solution of equations of the type of Eq. 6 for the buckling of columns with rotational restraints in structures of type (a), previously defined, is greatly simplified by the use of charts and nomograms, many of which have been prepared for this purpose. Fig. 9 is a typical illustration.

A very convenient simplification of the solution of Eq. 6 is also possible by use of the following approximation of Newmark for end fixity from the known rotational restraints, the error being less than 4%.

$$\frac{1}{k^2} \cong \left(\frac{\pi^2 + 4\,n_a}{\pi^2 + 2\,n_a}\right)\left(\frac{\pi^2 + 4\,n_b}{\pi^2 + 2\,n_b}\right) \quad \ldots\ldots\ldots\ldots \quad (9)$$

in which

$$n_a = R_a/(E_t\,I/L) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (10a)$$

$$n_b = R_b/(E_t\,I/L) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (10b)$$

and R_a and R_b are end rotational restraints.

A graphical solution for the buckling of a compression member elastically restrained by both rotational and translational restraints in structures of type

(b) defined above has been developed by Diedrichs[22]. In general this buckling condition is difficult to simplify and to generalize in graphical form except for specific types of structures, such as rigid frames. Solutions for various types of such structures will therefore be given in greater detail at a later point in this paper.

Recapitulation.—The treatment thus far has been perfectly general, and while quite satisfactory for analytical purposes, is perhaps too involved for specification use except on a broad basis. It must be understood that Eqs. 5, the Euler formula, and equations of the form of Eq. 6 and 7 covering the buckling of elastically restrained members, both are fundamentally identical. In Eq. 6 and 7, however, one deals directly with the numerical value of the

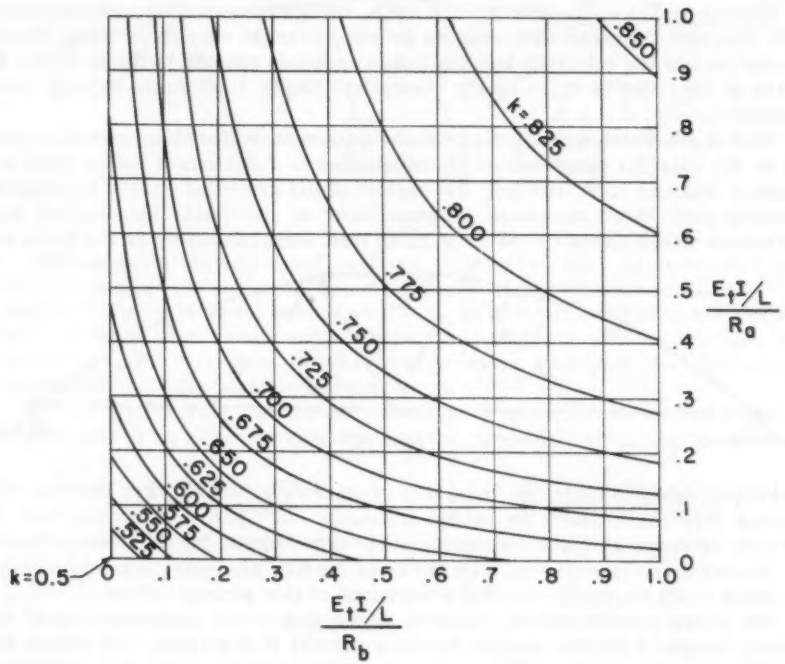


FIG. 9

restraints, and although the length factor k enters through substitution of its value from Eq. 5, the fundamental need for an evaluation of k and its subsequent use in Eq. 5 for design has been obviated, since by the procedure thus far presented the design takes place as part of the solution for Eq. 6 or 7. All axial loads considered in these equations are ultimate loads and the analysis becomes one of ultimate rather than working loads.

In general, therefore, and particularly since designers are accustomed and will continue to work with column formulas containing $\frac{k\,L}{r}$ terms, it appears preferable to utilize alternate procedures which give specific values for k for certain classes of members and configurations, for later use with such a column formula.

FRAMEWORKS WITH LITTLE OR NO TRANSLATION OF JOINTS

In triangulated truss frameworks, loads are usually applied to the joints themselves, producing axial loads in the members if the joints are hinged. Deflections of the members normal to their line of action are relatively small and are due to the axial deformations of the members under load. If the joints are rigidly connected, as by welding or by using heavily riveted gusset plates, some secondary bending is induced. The effect of these secondary distortions on the buckling of the truss framework is usually small and may be neglected in the buckling analysis. This materially simplifies the problem, which thus reduces to that of a column elastically restrained against rotation at both ends.

Chords of Trusses.—The most simple, though conservative, recommendation that can be made with respect to compression chords of truss frameworks is that the effective length of each member against buckling within the place of the truss be taken as the theoretical length L (distance between panel points); that is, $k = 1$.

Such a generalization disregards the restraints offered by web members or by the adjacent compression chord members. For trusses with a fixed live loading (such as roof trusses), the neglect of the restraint offered by adjacent compression chord members to each other is justifiable because all such members would approach their buckling load simultaneously as the loads are

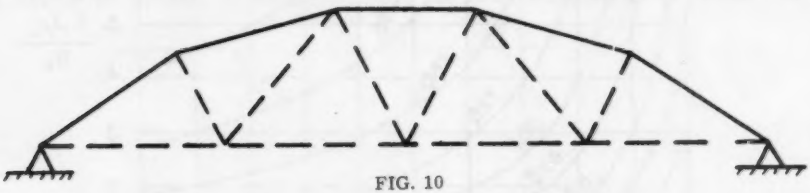


FIG. 10

increased, and are therefore incapable of offering restraint. For trusses with moving live loads (such as bridge trusses), such justification may not be strictly valid for all truss configurations; nevertheless, for the usual patterns of trusses there is sufficient similarity of loading producing maximum chord stresses in all chords to warrant acceptance of this generalization.

The above simplifications can also be applied to the determination of effective length of chords against buckling normal to the truss, and where the chord becomes a component of, say, an upper horizontal bracing truss system. Where a bracing system is lacking, as is the case with Pony trusses, the above approximations do not apply, and resort must be taken to procedures considering the buckling of the upper chord as a continuous column of elastic lateral supports.

Web Members.—If restraints of tension diagonals and compression chords be disregarded, compression diagonals may be considered as flexurally restrained against buckling in the plane of the truss by adjacent lower (tension) chord members (Fig. 11), yielding a tabular solution for k (Table 1) based on certain simplifying assumptions.[9]

A more general and conservative approximation resulting from the above is that web members be designed for a reduced length k L equal to 0.8 times the actual length.

For the case of crossed diagonal members of trusses (Fig. 12), the values in Table 2 for the effective-length factor k may be employed.[9]

A more general approximation of the values in Table 2 is to use a reduced length k L equal to 0.5 times the actual length of the diagonals.

For the case of the vertical web members in K-trusses (Fig. 13), with buckling perpendicular to the plane of the truss, the approximations (shown in Table 3) for k should be employed[8]:

Effects of Materials, Factor of Safety, etc.—Effective-length-design criteria are markedly affected by the type of material involved and by the varying requirements set by the specifications regarding factors of safety against failure.

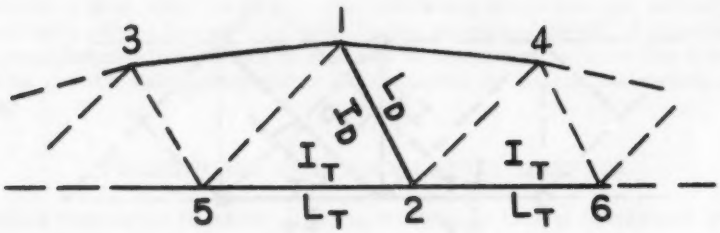


FIG. 11

With material having a well defined yield point, such as structural steel, there is a marked and rapid deterioration in the effective modulus E_t in the region of the yield point, beyond which E_t attains the value of zero. As a result of this, both compression members and tension members lose their ability to offer restraints to other compression members, and the effective length factor of the latter is reduced to unity.

It is a further fact that the deterioration of the effective modulus of structural steel is so great for columns with slenderness ratios less than about 60

TABLE 1.—WEB-MEMBER APPROXIMATIONS FOR k

$\frac{1}{6}\frac{L_T}{L_D}\frac{I_D}{I_T}$	k
0	0.700
0.02	0.714
0.04	0.724
0.06	0.738
0.08	0.749
0.10	0.760
0.12	0.770
0.14	0.779
0.16	0.788
0.18	0.797
0.20	0.805

TABLE 2.—CROSSED-DIAGONALS APPROXIMATIONS FOR k

$\frac{2}{3}\frac{(E\,I/L)\,T}{(E\,I/L)\,D}$	k
0	0.437
0.2	0.449
0.4	0.457
0.6	0.463
0.8	0.467
1.0	0.471
1.2	0.474
1.4	0.476
1.6	0.478

that the critical buckling load for a fixed column is only about one or two per cent different from that with pinned ends (assuming no translation). Thus, the introduction of k-values has little if any practical effect on the design of these columns in trusses. Similar remarks apply to low alloy and silicon steels. The difference in buckling loads of steel columns for the pinned and fixed end

conditions increases steadily as the slenderness ratio rises from 60 to 125, at which latter point the fixed-ended column load may approach 100% more than that of the pin-ended column. Thus the use of k-values results in appreciable economy in truss steel design for this slenderness ratio range.

All of the foregoing remarks apply only to the column without translation, since translation has a significant effect even for lower slenderness ratios.

With materials, like aluminum and magnesium alloys, the reduction in E_t is large as the yield stress is approached but E_t still remains finite, and the adjacent members retain some increase of capacity for offering restraints

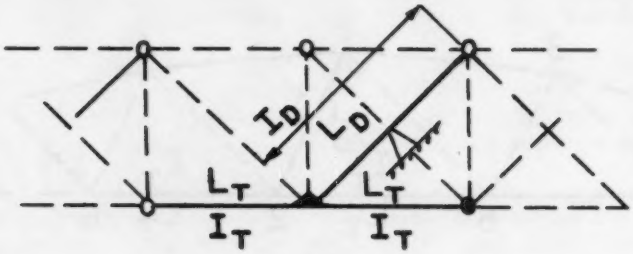


FIG. 12

even at high loads. Thus with such small slenderness ratios as 50, there is a sizeable difference between the buckling load of the pinned and fixed column, and the use of the proper k-value may have a significant effect for even these low slenderness ratios.

With most steel specifications a higher allowable stress is provided in tension than in compression, expressing the need for higher factors of safety for the latter type of action. Since tension members (if designed only by this criterion) would reach their yield point long before the compression members, the major restraints offered by the tension members disappear. Under

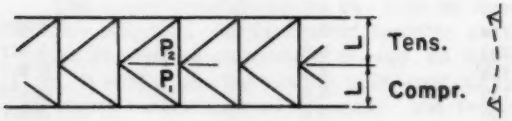


FIG. 13

TABLE 3.—k-VALUES FOR k-TRUSS WEBS

P_2/P_1	k
0	0.73
0.2	0.67
0.4	0.62
0.6	0.57
0.8	0.53
≥ 1.0	0.50

conditions where, for example, factors of safety in the range of 1.7 in tension and 2.2 in compression are employed, it appears desirable to design all members for k = 1 unless there is incorporated in the restraining tension members an excess of area for other reasons than to meet the stress needs (that is, minimum L/r requirements, inability to secure commercial sizes exactly equal to those theoretically needed, etc.)

Some generalizations are possible for the case of design for moving loads in steel by assuming uniform live and dead loadings. Assuming factors of safety of 2.25 in compression and 1.83 in tension, it may be shown that when

the dead load is zero, only those web members within the middle six-tenths of the span can possibly have end restraints from the tensile chords; and that this range decreases with increasing dead load until the dead load reaches about 1.68 times the live load, beyond which point the tensile chords can be counted upon to offer no restraints at all.

It is then apparent that for steel roof trusses and possibly some lightly loaded highway trusses it is quite possible that very little if any restraints will be offered by the tension chords to the web members, even in part-span loadings, with present factors of safety, unless over-design is resorted to.

Along the same lines while compression chords offer restraints to web members in part-span loadings, such restraints alone are not sufficient to reduce the k-values beyond 0.88 with present factors of safety. A combination with considerable tensile restraint would be necessary to bring the k values down to the 0.8 value given above as suggested by Bleich for web-member design.

FRAMEWORKS WITH TRANSLATION OF JOINTS

Where translation of joints, (that is, sideway or lateral movement) is possible, as in the case of rigid frames, the effective length of columns may generally be considerably larger than the actual length of the columns (Fig. 5).

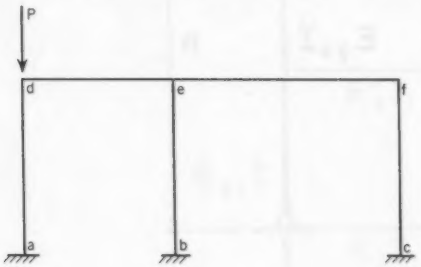


FIG. 14

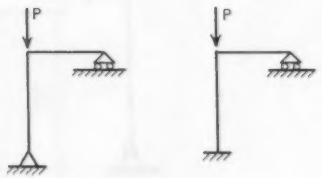


FIG. 15

Similarly, where free-standing columns occur, such as those supporting out-door crane runway girders and similar structures, a k-value of 2 has been previously noted as appropriate. It has been further noted in Fig. 5 that, theoretically, for some types of frameworks it is possible for the columns to approach an infinite effective length, although it can hardly be said that such types of structure are of a practical nature.

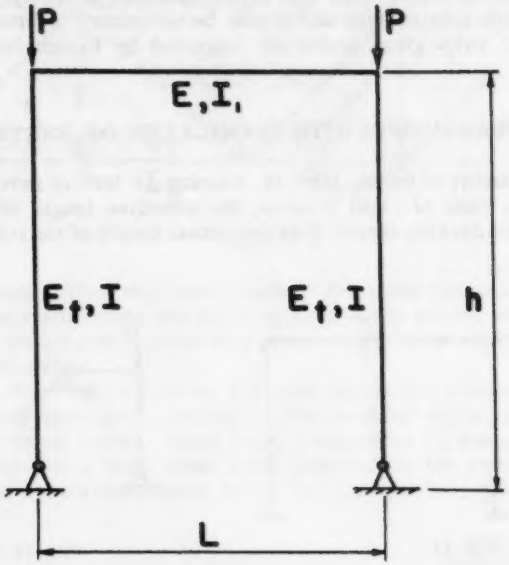
Direct Application of Eq. 7 for Column with End Restraints. —Eq. 4a covering the buckling load of a column with end rotational and translational restraints, can be applied for certain simple cases where the end restraints may be readily determined or approximated. Thus in Fig. 14, the column translational stiffness of be and cf can be conservatively stated as though the tops were pinned, and added together, to yield translational restraint; similarly, the rotational stiffness of de could be estimated conservatively as that with end e pinned; the resultant k-value from the charts would be conservative in all cases. Similarly, the cases in Fig. 15 are simply solved by Eq. 7.

For design purposes, however, the foregoing types of loading conditions usually are not critical, because all of the columns may be simultaneously under high stress, and the effects of the axial load would diminish their translational stiffness. Thus for the usual design purposes the analysis by Eq. 7

cumbersome and more appropriate stipulation would be to designate critical design k's for specific and common structural frames and loadings. These are covered below.

The solutions[8] shown in Figs. 16, 17, and 18 may be resorted to for the determination of k-values for the specific rigid-jointed frame loadings indicated.

Multi-Story Building Frames.—The two-story symmetrical rectangular frame of Fig. 18 is included here as representative of the type of approach followed by early classical investigators toward the general problem of multi-story building frame stability, in that constant column axial loads and constant stiffnesses of beams and columns were assumed in all stories.



$\frac{E_t\, I/h}{E\, I_1/L}$	=	0	0.1	0.2	0.5	1.0	5.0	∞
Held against translation,	k =	0.700	0.733	0.761	0.814	0.875	0.963	1.000
Free to translate,	k =	2.00	2.03	2.07	2.17	2.33	3.38	-

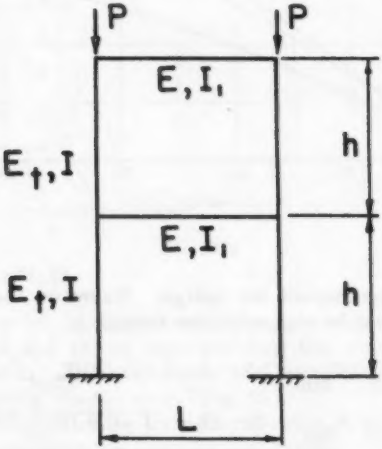
FIG. 16

Since these conditions do not exist in real structures, such a basis for analysis or design is not realistic but is of value mainly in providing some indication of the effect of the number of bays or stories in the structure.

Bleich[8], following these assumptions, developed the series of curves shown in Fig. 19 for the variation of effective length k vs. the ratio of column to beam stiffnesses, for a multi-story frame of n stories each of height h, and with an infinite number of equal bays laterally (Fig. 20). Each curve approaches asymptotically a value k = 2 n, indicating that the building column approaches a cantilever of n stories height as the beam stiffness decreases toward zero.

$\frac{E_t I/h}{E I_1/L}$	=	0	0.1	0.2	0.5	1.0	5.0	∞
Held against translation,	k =	0.500	0.524	0.545	0.590	0.626	0.680	0.700
Free to translate,	k =	1.000	1.016	1.030	1.082	1.156	1.501	2.000

FIG. 17



$\frac{E_t I/h}{E I_1/L}$	=	0	0.1	0.2	0.5	1.0	2.0	∞
Held against sway,	k =	0.500	0.507	0.668	0.689	0.753	0.803	0.879
Free to sway,	k =	1.000	1.033	1.065	1.160	1.310	1.515	4.000

FIG. 18

M. G. Puwein[26] followed a similar procedure. In general, this assumed a unit rectangular frame having equal columns and equal beams, free to sway as shown in Fig. 21. Using the rotational stiffness criterion for stability at the upper left joint[6] (namely, that when the frame is on the point of buckling, the sum of the rotational stiffnesses of the two heavily scored members is zero), one writes immediately:

$$(\pi/k)\tan(1/2)(\pi/k) = 6K_{bm}/K_{col} \quad \ldots\ldots\ldots\ldots (11)$$

This same result is obtained from the general equation (Eq. 7) for a column under known rotational and translational restraints at its ends.

Puwein also derived curves similar to those of Fig. 19, and went further to

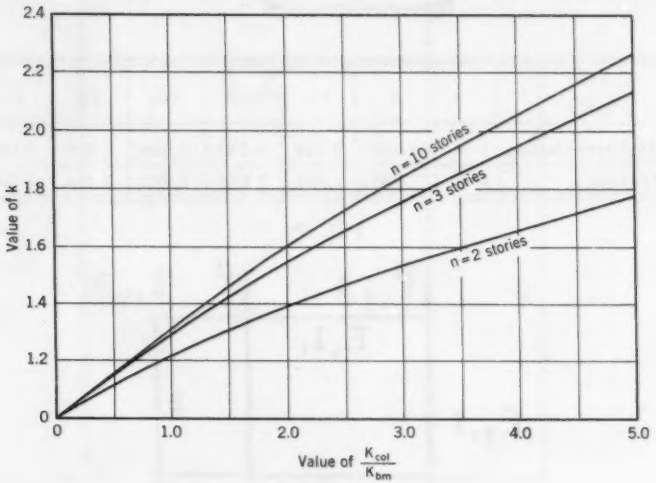


FIG. 19

develop approximations thereof for design. Based on the solution for an infinite number of stories, he suggested for design:

$$\text{For } K_{col}/K_{bm} < 1.2: \quad k = 1 + (1/3)(K_{col}/K_{bm}) \quad \ldots\ldots (12a)$$

$$\text{For } K_{col}/K_{bm} > 1.2: \quad k = \sqrt{1 + 0.8\,(K_{col}/K_{bm})} \quad \ldots\ldots (12b)$$

Another investigator, H. Sievers[27], proposed for multi-story frameworks of n stories:

$$k = 1 + (K_{col}/K_{bm})\left[\frac{4(n-1) + a1}{6n}\right] \quad \ldots\ldots\ldots\ldots (13)$$

Very recently, P. Sahmel[28], has reinvestigated and extended the Puwein procedure, deriving formulae specifically applicable to two-and-three-bay frames with varying number of stories.

Approximation formulas similar to those by the above investigators are found in the buckling specifications in use in Germany and Austria. It is of interest to note that the German specifications incorporate arbitrary correction factors for k when all columns are not fully loaded. Thus, in a two-

column bent, one column of which is heavily loaded by P, and the other by $P_1 < P$, a reduction factor $\sqrt{(1/2)(1 + m)}$ is applied to k, wherein $m = P_1/P$.

From a practical design standpoint the above types of formulas are not realistic in themselves since they yield a single k-value for all of the columns of the framework, due to the assumption of constant loads and moments of inertia in all columns. An important step forward was made by Puwein who suggested that the formulae be applied story by story, utilizing K_{col}/K_{bm} values which are an average of those at the upper and lower end of each column in each story (since the equations were based upon equal restraints at both ends). In this manner, Eqs. 12 become workable design tools.

The last word in the refinement of these concepts with relation to the effective length of columns in multi-story frames, has recently been developed by Julian and Lawrence[18], who have introduced improvements in the following respects:

1. Accuracy has been increased by taking account of differing rotational restraints at top and bottom of columns (instead of assuming these as equal, as has heretofore been done).

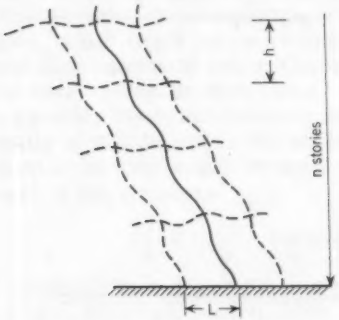


FIG. 20

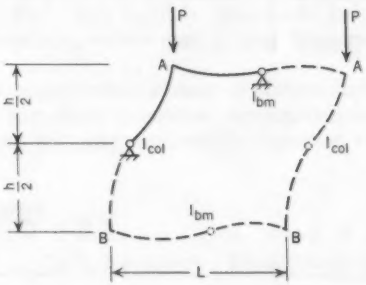


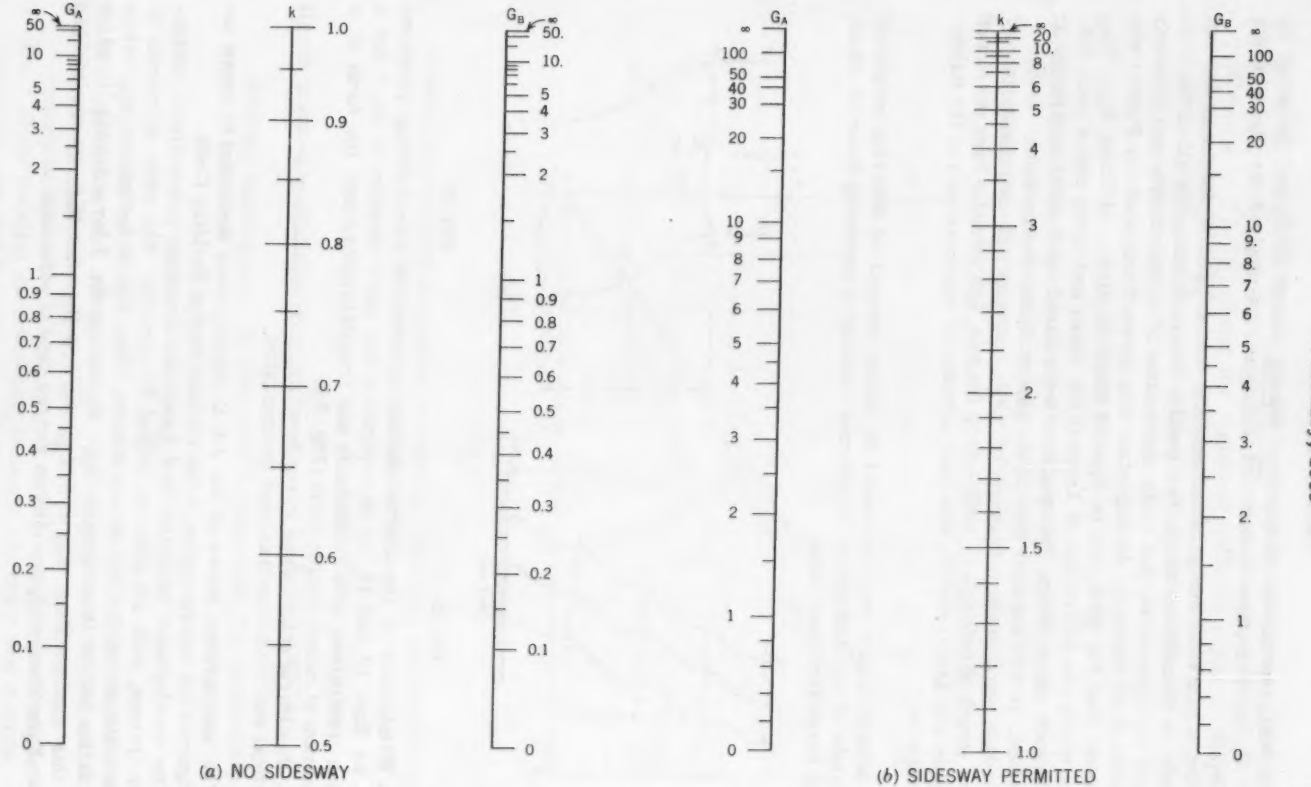
FIG. 21

2. Elimination of the approximations inherent in curve-fitting formulae such as Eqs. 12 and 13, by incorporating the exact solution of Eq. 7 for a column restrained both rotationally and translationally, into the form of a nomogram of rather simple form (Fig. 22).

3. By clearly establishing procedural rules, a satisfactory solution for all practical end conditions has been incorporated.

The nomograms, known as the J & L charts, were developed by these investigators for incorporation in the revised Boston Building Code.

The development by Julian and Lawrence assumes symmetrical rectangular frames, with all columns loaded to capacity. The ratio of column to beam stiffness at the top of any column (Fig. 21) is designated G_A, while that at the bottom is designated G_B. By solving Eq. 7 for sidesway, in which for this configuration $R_A = 6\,K_A$, $R_B = 6\,K_B$, $K = K_c(\pi/k)\cot(\pi/k)$, and $K'' = -K_c(\pi/k)\tan(\pi/k)$, one obtains for the case of sidesway:



(*a*) NO SIDESWAY (*b*) SIDESWAY PERMITTED

FIG. 22

$$\frac{\pi}{k}\cot\frac{\pi}{k} = \frac{G_A G_B \left(\frac{\pi}{K}\right)^2 - 36}{6(G_A + G_B)} \quad \ldots\ldots\ldots\ldots \quad (14)$$

Eq. 14 was used to plot the second nomograph (Fig. 22(b)). The first nomograph (Fig. 22a) covers the case of a frame with sidesway prevented, and was derived in similar manner using Eq. 7.

For the special case where $G_A = G_B$ in the sidesway condition, Eq. 14 reduces to the same form given in Eq. 6 used by earlier investigators.

For end conditions, Julian and Lawrence suggest that G be taken as equal to infinity for hinged ends, or for columns supported by but not rigidly connected to a footing or foundation. Where rigid attachment to a footing is provided, G is suggested as 2.0, but smaller values may be used if justified by analysis.

The J & L charts have been tested on many numerical examples and found yield results extremely close to the theoretical values. Their use, therefore, should be encouraged by practical designers.

Arbitrary fixed and length coefficients for certain classes of columns, such as are set up in the British Standards[23], may be too conservative for practical design. Overestimating k as 2.0 instead of its true value of 1.5, for example, would imply an overdesign of 78%. The British Standards coefficients omit important cases where k may be greater than 2, and thus may lead to under-design in such cases.

In general, the simplifications in this paper offer means of determining rationally a satisfactory reduced length for most practical configurations, which will not only result in safe designs but may materially improve the economy of the structure.

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DESIGN PRESSURE FOR A REACTOR CONTAINMENT VESSEL[a]

by John A. Bailey[1]

SYNOPSIS

The basis for selection of the magnitude of the reactor accident against which the containment vessel must provide protection is discussed and a limiting accident is analyzed. A numerical example is shown and generalized curves are included.

INTRODUCTION

Containment vessels are placed around water-cooled power reactors as a final barrier to the uncontrolled release of fission products, which could conceivably result from a severe reactor accident. While it is necessary to consider severe accidents in the design of containment vessels it should be noted that the precautions taken in design and operation make a containment-requiring accident very remote.

If a large amount of thermal energy is released and/or generated in a containment-requiring reactor accident and if there is no certain method of isolating the radioactive fission products, (both generally the case, at present) the containment vessel must also withstand the effects of this energy release or generation. One of the most obvious effects is the rise of internal pressure in the containment vessel.

The factors governing this pressure rise in water-cooled and moderated reactors are analyzed herein and a method is presented for calculating the enclosure pressures which could result from a reactor or primary coolant system

Note.—Discussion open until July 1, 1960. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Structural Division, Proceedings of the American Society of Civil Engineers, Vol. 86, No. ST 2, February, 1960.

a Presented at the May 1959 ASCE Convention in Cleveland, Ohio.

1 Atomic Power Equipment Department, General Electric Company, San Jose, California.

rupture accident. The design of the containment vessel is limited by the most severe accident for which it is required to design and so it is only the major accidents which will be considered.

THE PRESSURE-VOLUME RELATIONSHIP

Mechanism of Pressure Rise.—Since most water-cooled reactors operate at high pressures, a primary system rupture of any significant size would release liquid and/or vapor at a very rapid rate. Initially, the air around the break would be displaced by the escaping liquid and vapor. Mixing of air and vapor would follow shortly, with the two tending toward thermal equilibrium with each other. Some energy would probably be transferred to the air and vapor from the hot vessel or primary piping and some energy would be absorbed by cooler structural masses and equipment (including the containment-vessel wall). Any energy generated by chemical or nuclear processes would be released either during or after the accident. The peak internal pressure to which a given enclosure may be subjected is the result of the following competing rates of energy transfer into and out of the media within the enclosure:

1. Rate of energy release from the reactor vessel;

2. Rate of heat transfer to and from structural and equipment masses in the enclosure; and

3. Rate and timing of additional energy generation (chemical or nuclear).

Rate of Energy Release.—The rate of energy release from a primary system rupture would be a function of the size, location, and geometry of the break, the initial condition of the primary fluid, and the pressure into which the fluid discharges. For a given reactor and plant design the initial primary coolant conditions are confined to a relatively small range. The normal internal enclosure pressure is usually approximately at atmospheric level.

A break at a high elevation in the system would be more likely to permit partial vaporization of liquid within the vessel with the resulting discharge of a liquid-vapor mixture. A rupture near the bottom of the primary system would result in discharge of a mixture which would be primarily liquid. The equilibration of the liquid would probably occur mostly outside the reactor vessel or primary system. In the latter case the discharge rate per unit area of break would be higher. Rupture of a line connecting the reactor and steam drum or heat exchanger could permit the discharge from both sides of the break if it were a complete severance. A smooth rounded hole, such as that resulting from the severance of a vessel inlet pipe near a nozzle into the vessel, could have a higher discharge coefficient than a jagged square-edge break.

Perhaps the most difficult determination is that of the maximum break size. For a pipe break, obviously, there is a maximum flow area, but in the case of a vessel it is difficult to select the largest size primary system break which must be considered in containment design. It should be emphasized that the precautions taken in reactor and plant design and operation make any containment-requiring accident extremely remote. When considering cases of this degree of remoteness it is quite difficult to assess the "relative remoteness" of other occurrences which could have an effect on the accident.

One method of avoiding this problem is to assume, as has been done subsequently, that no energy is transferred into or out of the liquid, vapor, or gas during the process of equilibration. This makes the value of peak pressure independent of the discharge rate from the rupture.

Heat Transfer During Pressure Rise.—With weight rates of flow from a break in the region of 1,000 to 10,000 lb per sec per sq ft of flow area, the elapsed time between system rupture and peak pressure could, in the worst cases, conceivably be on the order of 1 sec or less. The inherent peak-pressure-reduction capability of a normal plant design could be negligibly small over this period of time. Moreover some considerable masses of metal are at temperatures above the final equilibrium temperature within the enclosure. Unless there is special equipment designed to reduce the pressure peak, it is questionable that any significant credit can be taken for this effect in any accident involving a large system rupture. Conversely any significant heat transfer from hot vessel walls or fuel during the pressure rise seems unlikely.

Additional Energy Contributions.—Should the accident be accompanied by a nuclear excursion there would be additional energy present in the system which could raise the peak pressure. A similar effect could be produced by any chemical reaction such as a metal-water reaction. A chemical reaction occurring after loss of water, for example, would probably not affect the first pressure peak but might cause a second peak whose magnitude would again be governed by the relative rates of energy transfer into and out of the thermodynamic media within the enclosure.

Dynamic Effects.—It is conceivable that a very rapid or nearly instantaneous rupture could generate pressure waves which could result in enclosure pressures above those calculated for thermal equilibrium. This could only be caused by an extremely rapid and large rupture which would seem unlikely in a reactor vessel and primary system in which the steel is generally in the temperature range where any failure would be of a ductile type. Moreover the primary system is usually surrounded by shielding material which would attenuate and partially contain any shock waves which might be generated.

An Analytical Model.—It can be seen that a rigorous analysis including all the possible variations in mechanisms and sequences would be a tremendously involved undertaking and when completed might not necessarily offer the degree of assurance required that the calculated peak pressure would not be exceeded in an actual accident. One way of avoiding the uncertainty of balancing the various energy transfer rates against each other is to make the conservative assumption that the equilibration of liquid, vapor, and air (with themselves) is accomplished in the complete absence of any energy losses. Any additional energy contributions which were concurrent with the rupture would be included. This would result in a conservative value of peak enclosure pressures. Following the peak pressure, energy losses could be included in the analysis as well as any delayed energy contributions. Any secondary pressure peaks could thus be calculated.

It is necessary to examine any secondary pressure peaks in order to establish their magnitude relative to the first peak. One type of accident which could possibly produce a secondary pressure peak higher than the initial peak predicted by the model, would be a rupture accident followed by a delayed metal-water reaction with all the energy being added only to the air and vapor. In this case the absence of heat absorption by vaporization of water could make the pressure-producing effect of the energy greater than in the case where water

is vaporized. The presence of massive shielding around the reactor greatly reduces the rate at which any subsequently generated heat could be transferred to the air and vapor. Because of this and the difficulty of getting any significant metal-water reaction, the model chosen is believed to be conservative.

ANALYSIS OF THE PRESSURE-VOLUME RELATIONSHIP

The following analyzes the specific situation of a rupture accident in which it is assumed that:

1. Equilibration of liquid, vapor, and air (with themselves) occurs adiabatically with respect to structural masses within the enclosure;

2. No additional energy contributions are made after initial equilibration;

3. There are no significant dynamic effects; and

4. Air follows the perfect-gas relation and has a constant heat capacity.

Air at low temperatures follows the perfect-gas relation so the amount of air in the enclosure, W_a, is

$$W_a = \frac{P_{a1} V_{a1}}{R_a T_{a1}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which P_{a1} and T_{a1} are the initial air pressure and temperature within the enclosure, R_a represents the gas constant, and V_{a1} is the enclosure free volume. The free volume is taken as that volume into which the air and water vapor are free to expand. If atmospheric moisture is considered, the partial pressure of the moisture, P_{m1}, and the amount of moisture, W_{m1}, may be found from an assumed value of initial relative humidity, ϕ:

$$P_{m1} = \phi P_{g1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which P_{g1} is the saturation pressure for water vapor at the chosen ambient air temperature, T_{a1}, and may be found in steam tables. The total internal enclosure pressure, P_{t1}, which is probably very close to atmospheric pressure is equal to the sum of the air and water vapor partial pressures, so the initial air pressure, P_{a1}, is given by

$$P_{a1} = P_{t1} - P_{m1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Assuming that the moisture behaves as though it were a perfect gas, the weight of moisture initially in the air, W_{m1}, is

$$W_{m1} = \frac{P_{m1} V_{a1}}{R_m T_{m1}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (4a)$$

or

$$W_{m1} = \frac{V_{a1}}{V_{m1}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4b)$$

in which

$$V_{m1} = \frac{R_m T_{m1}}{P_{m1}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4c)$$

The value of P_{m1} is found from Eq. 2 and $T_{m1} = T_{a1}$, an assumed value. As an alternate method, the initial moisture specific volume, V_{m1}, may be evaluated by

$$V_{m1} = \phi v_{g1} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

where ϕ is assumed, as in Eq. 2, and v_{g1} is the specific volume of saturated vapor at the initial ambient temperature, $T_{a1} = T_{m1}$. After the accident and thermal equilibrium, both air and water vapor occupy that portion of the enclosure and reactor vessel volume not filled with liquid water and the total internal enclosure pressure is the sum of the two partial pressures. It follows then that the sum of the liquid and vapor volumes is equal to the combined volumes of the reactor vessel and the enclosure free volume:

$$V_{w1} + V_{a1} = W_w (1-x) v_{f2} + (x W_x) V_{g2} \quad \ldots\ldots\ldots\ldots \quad (6)$$

in which W_w is the total amount of liquid and vapor (if any) within the primary system before the accident. That portion of the original contents of the primary system, W_w, which is vapor at equilibrium after the accident is x. There is no physical difference in the terminal condition, between moisture which was originally in the air, W_{m1}, and that portion of the primary system water which is vapor upon equilibration, $x W_w$. Solving Eq. 6 for the vaporized portion of the primary system water, x, gives the following:

$$x = \frac{\frac{W_{a1} + V_{w1}}{W_w} - \frac{W_{m1}}{W_w} (v_{g2} - v_{f2})}{v_{fg2}} \quad \ldots\ldots\ldots\ldots \quad (7)$$

Since $W_w = V_{w1} / v_{w1}$, where v_{w1} is the initial equivalent specific volume of the primary system fluid in whatever proportions the liquid and vapor may be mixed, and $W_{m1} = V_{a1} / v_{m1}$, Eq. 7 may be rewritten as

$$x = \frac{V_{w1}\left[\frac{V_{a1}}{V_{w1}}\left(1 - \frac{v_{g2}}{v_{m1}}\right)+1\right]}{v_{fg2}} = v_{f2} \quad \ldots\ldots\ldots \quad (8)$$

The initial primary fluid specific volume, v_{m1}, and initial enclosure moisture specific volume are functions of the assumed conditions before the accident. The terminal specific volumes (v_{f2}, v_{fg2}, and v_{g2}) are functions of the conditions existing after the accident. The ratio of enclosure free volume to primary system volume, V_{a1}/V_{w1}, is a design variable.

Since this idealized equivalent of an actual thermodynamic process is a non-flow, constant-volume process, the energy balance is written as

$$Q = U_2 - U_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

in which U_1 is the total internal energy of the media before the accident, U_2 denoted the corresponding value after release and equilibration of the primary water, and Q is the net energy contribution to the system from nuclear and/or chemical, heat generation, if any. Breaking down U_1 and U_2 into their component parts, Eq. 9 may be written as

$$Q = [W_w (u_{f2} + x\, u_{fg2}) + W_{m1}\, u_{g2} + W_a\, u_{a2}] - (W_w\, u_{w1} + W_{m1} + u_{m1} + W_a\, u_{a1}) \quad \ldots\ldots\ldots\ldots (10)$$

Note that u_{w1} is the equivalent specific internal energy of the entire primary system fluid in whatever proportions the liquid and vapor may be mixed. Rearranging and substituting

$$u_{a2} - u_{a1} = c_v (T_{a2} - T_{a1}) \quad \ldots\ldots\ldots\ldots\ldots (11)$$

$$Q = W_w (u_{f2} + x\, u_{fg2} - u_{w1}) + W_{m1} (u_{g2} - u_{m1}) + W_a\, C_v (T_{a2} - T_{a1}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

Using W_a (from Eq. 1), W_w, W_{m1}, and taking the original amount of energy in the primary fluid as

$$U_{w1} + W_w\, u_{w1} = \frac{V_{w1}}{V_{w1}}\, u_{w1} \quad \ldots\ldots\ldots\ldots\ldots (13)$$

Eq. 10 may be written as

$$\frac{Q}{U_{w1}} = \frac{V_{a1}}{V_{w1}} \frac{v_{w1}}{v_{m1}} \frac{u_{g2} - u_{m1}}{u_{w1}} + \frac{P_a\, v_{m1}\, c_v}{R_a\, u_{w1}} \frac{T_{a2}}{T_{a1}} - 1 + \frac{u_{f2} + x\, f_{g2}}{u_{w1}} - 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

Note that Eq. 14 applies as long as the amount of heat added does not exceed that which would cause vaporization of all the primary water; this is usually

the case. The volume occupied by air after the accident is very nearly equal to that before so

$$P_{a2} = \frac{T_{a2}}{T_{a1}} P_{a1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (15)$$

Eq. 15 neglects the slight increase in air volume resulting from the vaporization of some of the circulating water. Assuming thermal equilibrium, the terminal air temperature, T_{a2}, is equal to the saturation temperature corresponding to the vapor pressure within the enclosure after the accident. The total internal pressure, P_{t2}, is

$$P_{t2} = P_{a2} = P_{m2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

in which P_{m2} is the vapor partial pressure after equilibration.

After initial conditions have been assumed and a value placed on the net energy contribution, Q, there are five variables:

1. Final media temperatures, $T_{a2} = T_{m2}$;
2. Final vapor pressure, P_{m2};
3. Final air pressure, P_{a2};
4. Final total enclosure pressure, P_{t2}; and
5. Vaporized portion of the primary fluid, x.

There are five functional relationships:

(a) Eq. 8, x in terms of the final vapor pressure P_{m2} and temperature T_{m2} (through the steam-table values of v_{f2}, v_{fg2}, v_{g2});

(b) Eq. 14, x in terms of the final vapor pressure P_{m2} and temperature, $T_{m2} = T_{a2}$ (through steam table values of u_{f2}, u_{fg2}, and u_{g2});

(c) Eq. 15, final air pressure P_{a2} in terms of final air temperature, $T_{a2} = T_{m2}$;

(d) Steam tables, vapor partial pressure P_{m2} in terms of vapor temperature, T_{m2}; and

(e) Eq. 16, total enclosure pressure P_{t2} in terms of air and vapor partial pressures.

Because of the nature of the equations and functional relationship provided by steam tables, an explicit solution for P_{t2} and is not possible. It is necessary to assume a value of P_{m2} and solve Eqs. 8 and 14 for a value of x which results

in the proper value of Q/U_{w1}. Eqs. 15 and 16 yield the total internal enclosure pressure.

In order to generalize the analysis somewhat, Eq. 8 and 14 are arranged to use the ratio of enclosure free volume to primary system volume, V_{a1}/V_{w1}. For given initial conditions and net energy contribution, Q/U_{w1}, a curve of enclosure pressure, P_{t2}, and a function of V_{a1}/V_{w1} can be plotted.

The results of this analysis are shown in Figs. 1 and 2 assuming that the entire primary system is filled with saturated liquid. The internal enclosure

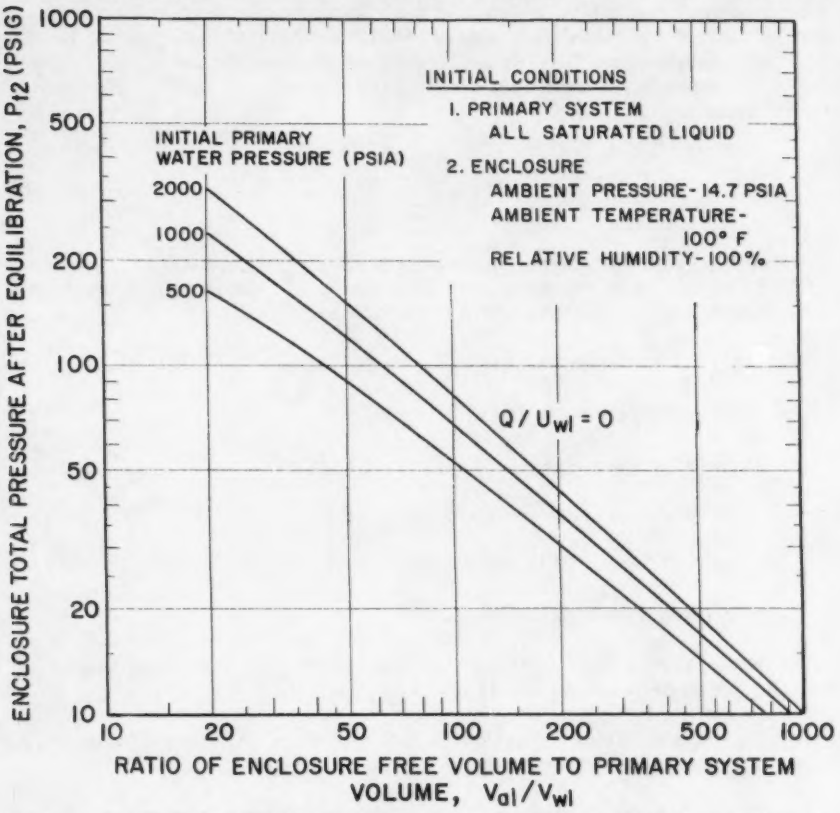

FIG. 1.—ENCLOSURE PEAK PRESSURE FOLLOWING PRIMARY SYSTEM RUPTURE, $Q/U_{w1} = 0$

pressure is shown as a function of initial primary system coolant conditions, the ratio of enclosure free volume to primary system volume, V_{a1}/V_{w1}, and the ratio of net energy contribution to initial internal energy, Q/U_{w1}. For a given initial primary coolant condition the plot of enclosure pressure, P_{t2}, against volume ratio, V_{a1}/V_{w1}, nearly linear on a logarithmic grid and, as expected, the relationship is an inverse one.

INFLUENCE OF THE PRESSURE-VOLUME RELATIONSHIP ON ENCLOSURE DESIGN

Generally a design criterion states that the enclosure design pressure must equal or exceed that peak pressure which could result from an accident of a given severity. The enclosure designer has then a certain latitude in balancing enclosure free volume against design pressure. On large containment vessels code restrictions and fabrication limitations may limit the acceptable range of

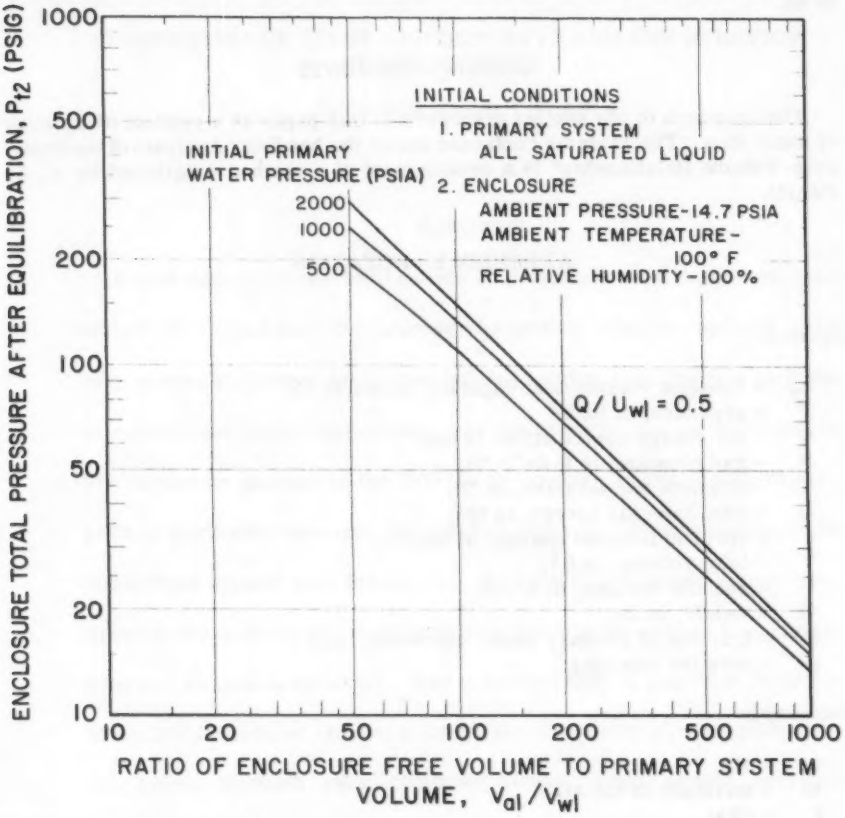

FIG. 2.—ENCLOSURE PEAK PRESSURE FOLLOWING PRIMARY SYSTEM RUPTURE, $Q/U_{w1} = 0.5$

volume and pressure values. Within these limitations the choice of enclosure size may be primarily an economic one.

FUTURE DEVELOPMENT

The expense of placing containment vessels around nuclear power plants is quite high. Considerable thought, analysis, and testing is being carried on to

find methods of reducing enclosure costs. One system now under consideration for reducing peak enclosure pressure encloses the reactor vessel in a chamber vented to the enclosure through a pool of water. The water would condense the escaping vapor, thus reducing the peak enclosure pressure. It has a further advantage of trapping a portion of fission products which might be released. The success of such a system in reducing the pressure peak depends on being able to place a reasonable limitation on break size, for there could be no such effect if the release of all of the primary coolant were instantaneous or nearly so.

ACKNOWLEDGMENTS

The approach to the subject discussed in this paper is a product of the work of many men. The analysis contained under the heading "Analysis of the Pressure-Volume Relationship" is a development of a method originated by C. A. Pursel.

APPENDIX I.—NOTATION

Symbols

c_v = constant volume heat capacity, in btu/lb °F;
P = pressure, in lb/ft^2
Q = net energy contribution, in btu;
R = gas constant, in ft lb/lb °R;
T = absolute temperature, in °R;
U = total internal energy, in btu;
u = specific internal energy, in btu/lb;
V = total volume, in ft^3;
v = specific volume, in ft^3/lb;
W = weight, in lb;
x = fraction of primary water vaporized; and
ϕ = relative humidity.

Subscripts

a = air;
m = moisture in the air;
t = total;
w = primary water;
1 = initial condition;
2 = terminal condition;
f = property of the saturated liquid;
g = property of the saturated vapor; and
fg = property of change upon vaporization.

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PROPERTIES OF STEEL AND CONCRETE AND THE BEHAVIOR OF STRUCTURES[a]

By George Winter,[1] F. ASCE

SYNOPSIS

A vast amount of new information on performance of structural materials under load is in the process of affecting, in a far-reaching manner, many established design procedures, and of creating new methods to replace outmoded approaches. At the present rate of development few engineers manage to familiarize themselves in detail with the results of pertinent research. Yet most of these new departures cannot be understood without such familiarity. The aim of this paper is to review, broadly, the basic features of these new findings on materials performance and structural behavior. They relate not only to plastic or inelastic materials properties, but also to brittle fracture, fatigue, residual stresses, creep, strength under combined stress, actual versus specified strength, and the like. Each of these features is presented not by itself,

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[a] Presented, by invitation, at the February 1959 ASCE Convention in Los Angeles, Calif. to introduce a symposium on plasticity in structural design.

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but in relation to overall performance of structures of the civil engineering type. None of the quoted research is of the writer's creation, and the only originality that can be claimed is one of presentation and interpretation.

INTRODUCTION

In his early training every structural engineer has become acquainted with the basic tools of his art in a course known as "Strength of Materials". This designation is as complete a misnomer as one will ever find in technical literature. In fact, most of the subject matter of "Strength of Materials" has little connection either with strength or with materials. It might more appropriately be called "Stresses and Deformations of Ideally Elastic Members." This describes, in brief, that conventional approach to structural design which has dominated established engineering practice until very recently. It consists, in essence, of computing stresses at design loads on the basis of assumed elasticity, and in implying that these stresses, in a manner rarely stated explicitly, have some relation to the strength and performance of the structure.

The revolution now under way in many fields of structural engineering is not correctly described by the juxtaposition of the terms "plastic" and "elastic." It is much more aptly characterized by the concepts of "strength" versus "stress." This revolution, in short, consists in the recognition that to be optimal in regard to safety and economy, design must be based on rational methods for predicting the actual strength of the structure (so that design loads can be held below that strength by an adequate margin), supplemented by means of predicting performance at service loads (so that impairment of serviceability by excessive deformation, vibration, etc., can be prevented.) Wherever it is the plastic properties of a material which govern the strength of a structure, "plastic analysis or design" is in the process of replacing the conventional "elastic" methods. Where other features determine strength and performance, attention is increasingly focused on them.

Among these other features are: Brittle fracture of normally ductile materials; the strength, brittle or ductile, of materials subject to bi- and tri-axial conditions of stress; creep as it affects not so much the strength as the serviceability of some types of structures; fatigue of structures subject to large numbers of not necessarily cyclic load repetitions. In addition, increasing attention is being paid to the actual properties of structural materials "as received" (as distinct from their stipulated, nominal minimum properties) and on the statistical distribution of these actual properties as a guide for a realistic assessment of structural safety. The methods evolving from all these tendencies differ increasingly from the conventional approach of comparing an elastically computed stress with a stipulated fraction of a specified, nominal materials property.

And yet, the supercilious disparagement of conventional "elastic design," fashionable among some of the more enthusiastic exponents of the "plastic" methods, is misplaced and unjustified. The Eiffel Tower, the Golden Gate Bridge, the Empire State Building, the Sando Bridge in Sweden, the flowering

of shell construction, the rigid framing of modern power stations, harbor sheds, and hangars, are living monuments to the power of elastic theory as a scientific basis for the art of design and construction. It has long been part of this art to temper the formal results of elastic computation by judgment, rational or intuitive, in order to reflect more realistically experience with actual performance. Much of what we are trying to do now is merely to incorporate into the science of design in an explicit and rational manner those very same features; to take them out of the realm of the qualitative and intuitive, and to put them on a sounder, quantitative, and scientific basis. No matter how far we go in this direction, an element of the art will always remain part of structural engineering.

It is the intent of the writer to review the most important of these developments. The tremendous scope of the relevant material imposes the character of this brief review which, of necessity, is one in breadth rather than in depth.

STEEL

Stress-Strain Curve.—Much of the information on structural performance of ductile metals is contained in the stress-strain diagram. Only to the extent that the stress-strain curve becomes known and reproducible is it possible to develop design procedures of high accuracy and reliability. For some structural metals, such as aluminum alloys, control and predictability of stress-strain curves has been achieved only lately.(1)[1] For others, such as stainless steel Type 301, otherwise a structural metal of high quality, the fact that adequate control of the effective stress-strain curve seems not yet in hand appears to represent the main difficulty in developing reliable design procedures.(2)

In contrast, for tension-test specimens of mild structural carbon steels the general character of the stress-strain curve has long been known to be that of Fig. 1. In view of the extreme elongations preceeding fracture, it is only the initial portion of the curve which is of primary structural interest (Fig. 1(b)). For most simple applications it is sufficiently accurate to replace this portion by the idealized elasto-plastic diagram of Fig. 1(c). However, in connection with inelastic buckling of steel members, which is of particular importance for plastic steel design, the initial strain-hardening range and the strain-hardening modulus for that range, E_{st}, have become increasingly consequential and have been extensively investigated.(3)

Yield Point.—The single, most important structural property of a mild steel is its yield point. Except where buckling enters the picture, allowable stresses in conventional design depend almost exclusively on this property. Likewise, carrying capacity as defined in plastic design is generally directly proportional to the yield point, again barring the influence of buckling. It is essential, therefore, that this quantity be well defined and be known with an adequate degree of certainty.

Routinely, yield points are determined in mill tests on specimens cut from the webs (for I-shaped sections). These tests are made at relatively high strain rates and by ASTM definition the yield point is customarily determined by the "drop of the beam or halt of the gage" method. By the nature of this method it is the upper yield point which is so determined. However, this upper yield point is an unstable phenomenon and structural strength is generally governed by the stable, lower yield point. Here, then, is one influence which makes for

[1] Numbers in parantheses thus, (1)—refer to the appended Bibliography.

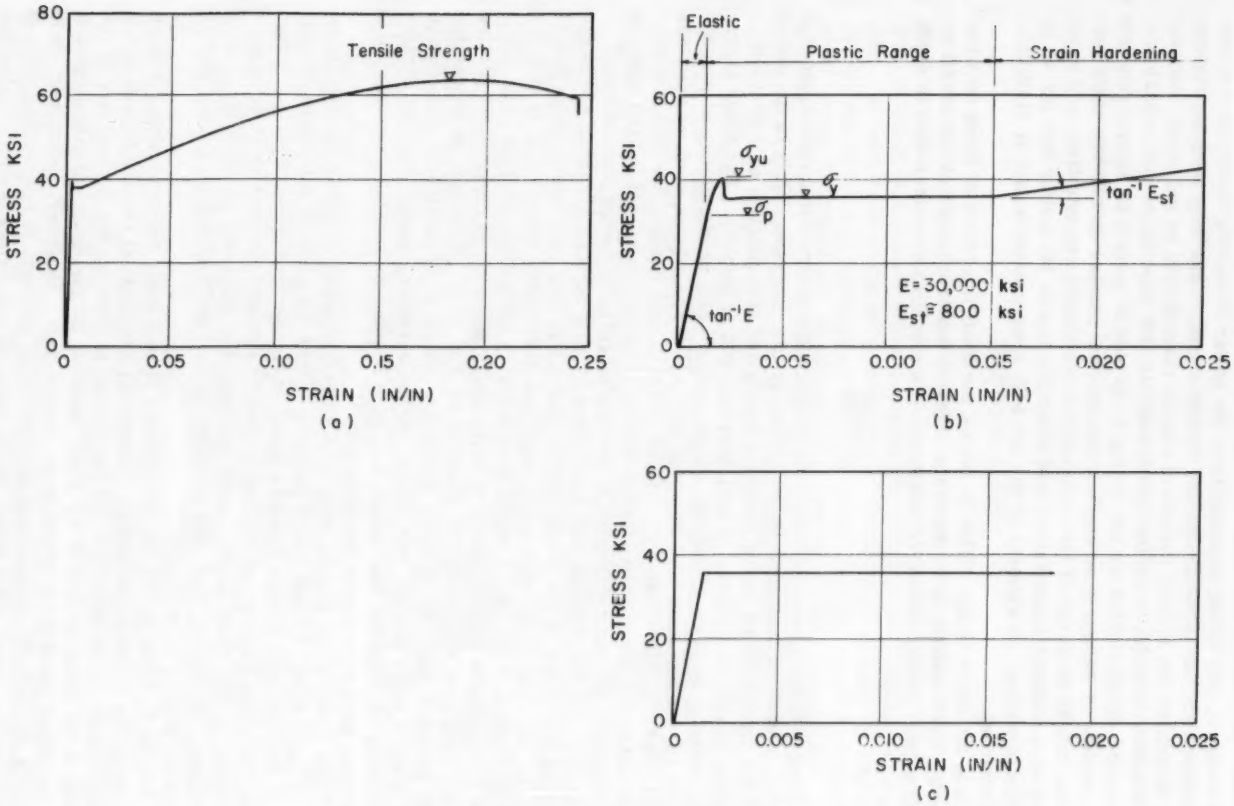


FIG. 1.—STRESS-STRAIN CURVE OF STRUCTURAL STEEL

the fact that the actual value of this structurally important porperty is below that given in mill tests. A further influence, tending in the same direction, is the fact that the relatively high strain rates (rates of loading) used in mill tests are significantly faster than the very slow rates of essentially static loading which take place in most structures. Since the yield point of steel increases with increasing strain rate, the values measured in mill tests are higher than those which are effective in the structure. Lastly, it is generally observed that if specimens are cut from the thicker flanges rather than the thinner webs, their yield points are lower than those of the web specimens which are used for mill tests. Each of these several influences rarely exceeds 10%, but since they all tend in the same direction it can be said that actual effective yield points of rolled steel members are from 5% to as much as 30% below those

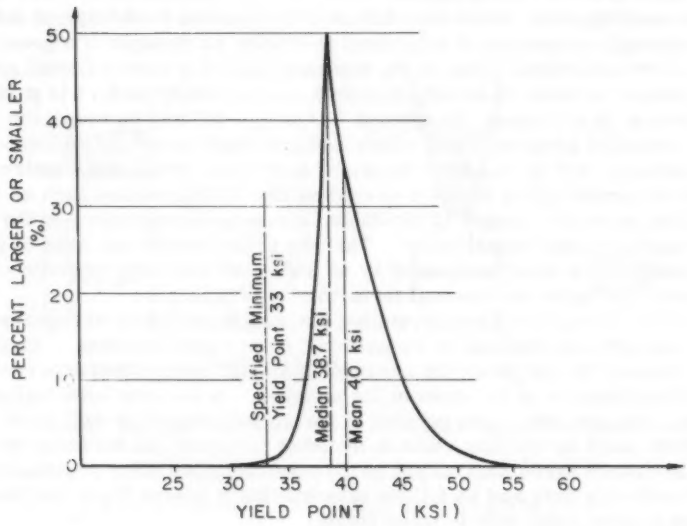


FIG. 2.—PROBABILITY CURVE OF MILL TESTS FOR STEEL (ASTM A-7)

indicated by mill tests.(4) It follows that mill tests should show yield points about 20% larger than the stipulated values on which strength calculations are based, or, conversely, that for computations of strength a yield value should be used which is about 20% below that indicated by mill tests.

Steel mills conduct their operations in such a manner that only a negligible fraction of their steels will fail to meet the specified minimum yield point as determined by mill test. To ensure this, production is so arranged that the average yield stress is considerably above the specified minimum. The probability curve of Fig. 2 reflects this clearly. It shows results of 3,974 mill tests representing 33,000 tons of structural steel (ASTM A-7) on nine projects between 1938 and 1951.(5) For this steel the specified minimum yield point is 33 ksi. It is seen that the median mill test value was 38.7 ksi, with a mean value of 40 ksi, and that less than 2% of the mill tests failed to meet the 33-ksi

requirement. However, if one considers that the structurally effective actual yield point is on the average only about 80% of that indicated by mill test, then, in order to ensure that the actual yield point be at least 33 ksi, the mill tests should show not less than about 41 ksi. It is seen that this is in the range of the mean rather than the minimum values of large samples of actual mill tests. It follows that about half of the steel rolled to ASTM A-7 requirements seems to have structurally effective, static yield points less than 33 ksi, even though only a negligible fraction will fail to pass the mill-test requirements. The amount of this deficiency will seldom exceed 15% although it does reach about 25% in small fractions of the total sample. It follows that if plastic design continues to be based on the specified minimum yield point, and if this yield point continues to be determined by present mill-test procedures, then the discrepancies which have been discussed represent one of the features which need to be taken care of in the safety factor.

The investigations which have led to this improved knowledge of the actual yield-strength properties of structural steels are an example of a general tendency of recent origin. This is the tendency toward a more rational approach to the entire question of structural safety, and to safety factors in particular. The attempt is to reduce the amount of "judgment" and increase the factual and quantitative basis on which safety and load factors are determined. While it is unlikely that it will ever be possible to deal with safety entirely on a statistical-probabilistic basis, it is evident that if information such as the one just cited on yield strength is available, it can be incorporated in the safety factor explicitly and numerically. Thereby it decreases that area of general uncertainty which must be covered by an undefined amount of reserve strength and which had led to the ironical term "factor of ignorance."

Residual Stresses.—The information discussed previously was obtained from materials tests carried out on coupons cut from rolled sections. Until relatively recently it was generally assumed that such coupon tests correctly reflected the behavior of the steel in the member. It has now been learned that residual stresses which are present in hot-rolled shapes as well as in members fabricated by welding, have a decisive influence on buckling strength. There is no such agreement, as yet, on the possible influence of residual stresses on brittle fracture and on fatigue strength, but it seems likely that they will be found to play some role in these fields.

Residual stresses in steel members arise from two sources—from differential cooling and from cold working, such as cold straightening. From the very nature of these processes there is a sizeable degree of randomness involved in the magnitude as well as in the distribution of residual stresses. However, at least in rolled shapes, the cooling stresses are by now quite well known both as to order of magnitude and types of distribution.(6)

It is easily realized that if a hot-rolled member cools down from the partially plastic state, those portions which cool first and are fully solidified while other portions are still hot, partially plastic, and contracting, will contain residual compression stresses when the member is fully cooled. Conversely, those portions which cool last find their tendency to contract counteracted by those adjacent portions which are at lower temperature; they are, therefore, subject to residual tension stresses, when fully cooled. In view of this, it is easily understood that the distribution of residual stresses in wide-flange shapes varies between the two extremes shown in Fig. 3. In the stocky, thick-walled, approximately square sections generally used for columns (Fig. 3(a)) the flange tips cool first, the web lags behind the outer portions of the flanges,

and the junctions of web and flange are the last to cool. This leads to high residual compression stresses in the outer parts of the flanges, and to residual tension stresses throughout the web. Conversely, in deep and relatively thin sections as they are mostly used for beams (Fig. 3(b)) cooling begins in the central portion of the web as well as at the flange tips, while the last parts to cool, as before, are the web-flange junctions. Correspondingly, high residual compression stresses appear in the web as well as in the outer flange portions, while residual tension is restricted to the inner portions of the flanges and the immediately adjacent parts of the webs. Very extensive measurements on large numbers of specimens have shown that a good average value for the maximum residual compression stresses is 13 ksi, with individual values occasionally reaching as much as 20 ksi. There seems to be evidence that these same values apply not only to low carbon, but also to high-strength, low-alloy steels.

It is well known that under ductile conditions internal, residual stresses do not influence the static strength of members, no matter what their magnitude

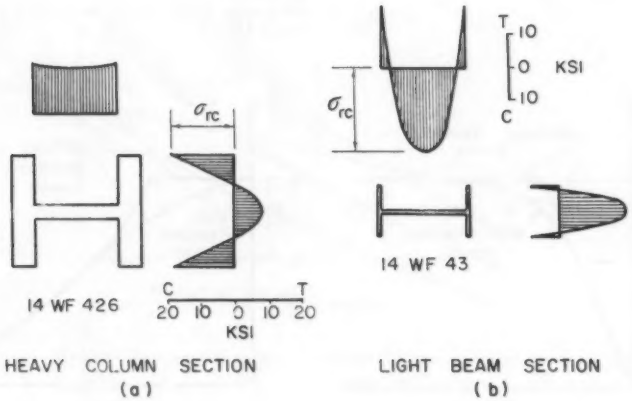


FIG. 3.—RESIDUAL STRESSES IN ROLLED SHAPES

and distribution. This is so because residual stresses are of necessity self-equilibrating, that is, have zero force and zero moment resultant. Therefore they neither add to nor subtract from moments and forces caused by external loads. However, they can be a factor in reducing resistance to brittle fracture and to fatigue. In addition, they change the load-deformation characteristics of the member and, therefore, can decisively affect its buckling strength.

If a short piece of the column shape of Fig. 3(a) is concentrically compressed, the applied uniform stress P/A superposes on the residual stresses. Yielding will start at the flange tips when the applied plus the residual stress equals the yield point:

$$\frac{P}{A} + \sigma_{rc} = \sigma_y \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (1)$$

As the load is further increased, yielding spreads inward and additional load is resisted only by that shrinking part of the cross-sectional area which is still elastic. It is easily verified by elementary computation that the resulting

load-deformation diagram is curved. If one plots it in the form of a stress-strain diagram, it has the shape of Fig. 4(a). On this same figure is drawn the simplified (elasto-plastic (see Fig. 1)) stress-strain diagram which would be measured if no residual stresses were present. It is seen that their effect is to produce in the steel member an "artificial" proportional limit

$$\sigma_p = \sigma_y - \sigma_{rc} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

equal to that applied stress, P/A, at which yielding starts in the flange tips.

On the other hand it is known that when a column is concentrically compressed to a stress beyond the proportional limit it buckles at the load

$$P_{cr} = \frac{\pi^2 E_t I}{L^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

or at the stress

$$\sigma_{cr} = \frac{\pi^2 E_t}{(L/r)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

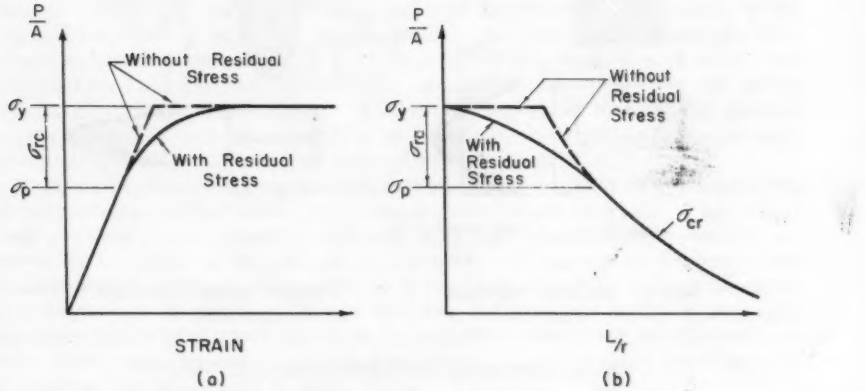


FIG. 4.—INFLUENCE OF RESIDUAL STRESS σ_{rc} ON COLUMN STRENGTH

in which E_t is the slope of the tangent to the stress-strain curve (dashed line in Fig. 4(a)). Eq. (4) is plotted on Fig. 4(b) together with that curve which would result if no residual stresses were present. It is seen that in the practically most important range (up to about L/r = 120 in steel) the presence of residual stresses reduces the buckling strength of columns very significantly.(6,7) The same holds true for lateral buckling of beams and, to a lesser extent, for local buckling of compressed flanges and webs.(3)

This large influence of residual stresses on buckling strength calls for adequate information on their magnitude also for members other than hot-rolled shapes. In regard to members fabricated by welding there are some indications that residual stresses may be even larger than in rolled shapes. Corresponding investigations are now under way at Lehigh University.

Brittle Fracture.—Ductile metals, and particularly steel, under certain conditions will fracture in a brittle manner, suddenly, and without any warning by preceeding plastic deformation or distortion. A considerable number of more or less catastrophic failures, not only of ships, but also of bridges, storage and pressure vessels, and other structures have been traced to this cause.

The nature of brittle fracture is most easily understood in the following manner: Two types of strengths can be distinguished in ductile materials, namely that stress which will cause sliding (yielding) through a shear-type displacement, and that stress which will cause separation (cleavage) by a sudden break without noticeable plastic deformation. If the yield strength is lower than the cleavage strength then, as the structure is loaded, yielding will occur long before cleavage fracture and ordinary, ductile behavior is obtained. The important point is that neither the yield strength nor the cleavage strength of a material are constant but change in different ways depending on a variety

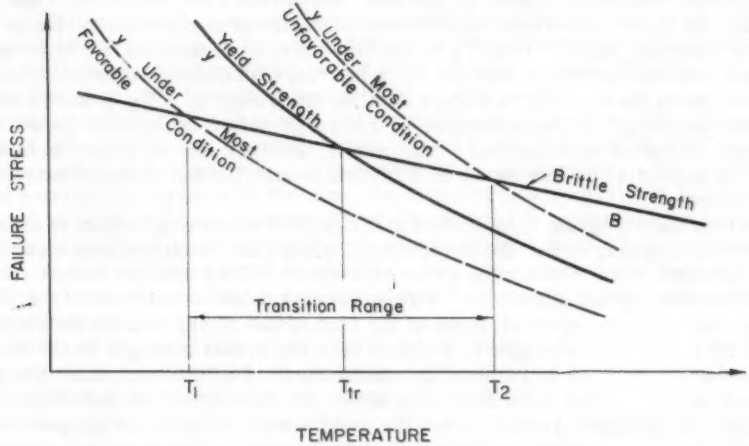


FIG. 5.—SCHEMATIC EXPLANATION OF BRITTLE FRACTURE

of influences. Among these, temperature is one of the chief factors and the nature of brittle fracture is most easily visualized by a plot of the two types of strength versus temperature.

The solid curves of Fig. 5 show, schematically, the manner in which the ductile yield strength Y and the brittle cleavage strength B depend on temperature. It is seen that both decrease with increasing temperature, but that Y decreases at a much faster rate than B. If a structure subject to increasing load is at a temperature larger than that marked T_{tr}, then, as the load is increased the stress will reach the yield strength Y before it can attain the cleavage strength B. Consequently, yielding will start and the structure will behave in a desirable, ductile manner. Conversely, if the service temperature is lower than T_{tr} then, with increasing load, the cleavage strength B will be reached first, and sudden, brittle fracture without prior yielding will ensue. For this reason T_{tr} is known as the transition temperature, below which behavior is brittle and above which it is ductile.

Temperature, however, is not the only variable. While the cleavage strength is relatively insensitive to other influences, the yield strength, particularly of steels, depends on a number of factors. It is the greater the higher the strain rate, the larger the carbon content, the coarser the grain, the higher such stress concentrations as may exist, and it is also increased by the presence of triaxial tension stresses, such as residual stresses from welding.(8) On the other hand, the yield strength is the lower the finer the grain size, the higher the manganese and the lower the carbon content, the slower the strain rate, and the more uniform and nearly uni-axial the stress field.(9) Assuming, for simplicity, that the brittle strength on the other hand, is not influenced at all by these factors (which is only an approximation), it is seen that even for a single steel of given chemical composition the yield strength can vary over a wide range depending on the degree of simultaneous effect of the various enumerated factors. This is shown schematically by the two dashed curves in Fig. 5. The left one of these indicates the magnitude of Y under the most favorable conditions (small grain size, low strain rate, etc.), the right one under the most unfavorable conditions (high stress concentration, large triaxial tensions, etc.). Then T_1 is the "nil ductility temperature" at or below which fracture is always brittle; T_2 is the "nil brittleness temperature" at or above which the fracture is always ductile. The range T_1 - T_2 is known as the transition range. At temperatures in that range, which comprises the service temperatures of most ordinary structures, fracture can be either brittle or ductile depending on the degree of unfavorable effect of the various enumerated influences.

From the foregoing it is evident that brittle fractures can occur at average stresses sizeably below the conventional yield point if stress concentrations are present, since these have a two-fold effect: They produce a peak stress k-times the average stress (k = stress-concentration factor) and at the same time they raise the yield strength at the root of the stress raiser. Hence if, at that root, the yield strength Y is higher than the brittle strength B, the stress concentration cannot be relieved by plastic flow. If at the same time the peak stress at the root exceeds B, then a brittle fracture starts at that root, even though the average stress is below the yield stress of the metal proper. Once started, such a crack will generally, though not always, propagate because very little energy is needed to keep a brittle fracture going, and because the speed of the crack propagation produces a high strain rate which, by itself, is an unfavorable influence.

It is seen from this much simplified description that the importance of brittle fracture is by no means restricted to impact conditions. In fact, a number of ships have suffered brittle failure while at rest in harbor, and a number of bridges while free of any significant live load. It is also seen that in welded construction, particularly, a really high degree of safety against brittle fracture can be achieved only through a combination of (a) very careful detailing (to reduce to a minimum all stress concentrations), (b) a maximum of weld inspection (to detect and eliminate weld flaws), (c) appropriate weld layout and welding procedure, including preheating and postheating (to reduce triaxial tensions) and (d) the use of special steels of low notch sensitivity (several of which have recently been introduced). All these measures are costly and for this reason can be used only to a limited degree in the usual civil engineering structures.

It is not certain to what extent this situation has relevance to the newly developed methods of plastic steel design, particularly of outdoor structures.

Evidently, the premises of plastic design are satisfied only if sizeable plastic deformations at yield-stress level can actually take place before cleavage fracture occurs. Laboratory tests on full-size frames at indoor temperatures seem to indicate no danger of premature brittle fracture (that is, of brittle fracture prior to the formation of the last yield hinge). However, it should not be forgotten that fabrication, welding, and testing of these frames have been carried out under optimum laboratory conditions. Also, the structures tested were essentially two dimensional, with no framing members connected by welding at right angles to the plane of the structure. Actually, stress concentrations and tri-axial tension are likely to be worst at those many points of actual structures where members in three mutually perpendicular directions are joined. Experienced welding engineers, therefore, have expressed doubt "whether there is always a predictable amount of plastic action or reserve strength in structural steel" and that, unless and until this is proved "the possibility that here may be no *(or insufficient)* plastic action must be recognized and the designs of structures planned accordingly" (Mr. F. H. Dill, quoted in (10)). This quotation is not intended to cast doubt on the soundness of plastic design in most situations. It is intended to indicate an area where influence of material's behavior of one kind on a design method based on behavior of another kind may need further investigation and safeguards.

Fatigue.—The nature of fatigue and the multitude of factors which influence fatigue life are a matter of such complexity and scope that even an abbreviated treatment within the frame of this paper would be meaningless. The problem is of extreme importance in the case of machinery and generally of structures subject to large numbers of wide stress fluctuations or reversals, of uniform or nearly uniform peak intensity. It is evident that most civil engineering structures do not answer this description. For this reason the writer will restrict his remarks on fatigue to a few suggestions regarding the relevance of fatigue to buildings and bridges.

Laboratory fatigue tests on structural members and connections, carried out between fixed limits of either complete reversal or of zero-maximum-zero cycles often show alarmingly low fatigue strength. This is particularly true for many types of welded connections. With the more general availability of laboratory pulsators, the fatigue testing of entire structures, or at least structural subassemblies, has become possible. In fact, in certain European countries the writer has observed a tendency summarily to substitute fatigue testing by pulsators for static testing, even for such assemblies which, in actual use, would never be subject to fatigue loading of any severity, such as roof and floor structures. Such testing is certainly spectacular in showing up places of high stress concentration and the like, but its relevance to actual structural performance does not seem to have been established.

R. Weck has summarized his judgment as follows:(11)

> "The risk of fatigue failure is entirely absent in buildings unless there are travelling cranes. Even then the number of times the crane is used to full capacity may not be large enough to affect the gantry. Wind loading is variable and produces stress fluctuations but neither their number nor their magnitude is large enough for fatigue failure to result. (However) wind may lead to fatigue failure if certain parts of a structure are excited to resonance vibrations by the action of the wind. In highway bridges those stresses due to live load which occur a very large number of times are generally only fractions of the full design stress and in

general will not result in fatigue failure—although this depends on detail design—whereas the full design stress is not reached frequently enough during the life of the structure to cause trouble. The situation in railway bridges is very different."

The statement on highway bridges, based on European conditions, may need some modification in the United States in the direction of greater caution, in view of the higher frequency of heavy truck loads in our highway traffic as compared to that in Europe.

There is a fundamental difference between the fatigue regime of such things as crankshafts, connecting rods, cranes, presses, aircraft structures, etc. on the one hand and buildings, highway bridges, and the like on the other. In the former most stress cycles are of maximum or near maximum intensity and, therefore, fatigue tests of the ordinary kind are reasonably representative. In the latter, while large numbers of stress cycles may occur, only a small fraction of them will be of, or near, maximum intensity. The application to such structures of test results obtained with cycling between fixed stress limits is, therefore, misleading and excessively conservative.

The experimental exploration of fatigue strength under the kind of variable cycling which actually occurs in structures is only in its infancy. The difficulty of reproducing in the laboratory cycling of a nature statistically similar to that even in a railway bridge are considerable, and become almost insuperable for other more random types of loading, such as wind. The authors of one of the few extensive investigations in this field[12] summarize: "From the limited number of tests it may be concluded that the variations in maximum cyclic stress obtained in railway bridges may be expected to provide a life markedly greater than one would expect to obtain on the basis of the maximum applied stresses." They estimate this increase in life to be "possibly equivalent to a decrease of approximately 2,000 - 3,000 psi in the maximum stress in constant-cycle loading." Since maximum-stress cycling is more nearly approached in railroad bridges than in practically any other structures including most cranes (with the exception of certain cranes in steel mills whose loads vary within narrow limits), these experimental results tend to confirm what was quoted in regard to low fatigue danger in buildings and bridges.

There are situations, however, where a combination of fatigue and brittle fracture may result in serious consequences when each of these occurrences, by itself, would not have been cause for alarm. Fatigue cracks are initially of microscopic proportions. At the slow rate of cycling prevalent in most structures (as distinct from high-speed machinery) they progress very slowly and may take many months and even years to reach such proportions that they will fail the member. With normal inspection and maintenance they are usually detected and corrected in time. However, if such minute fatigue cracking occurs below the transition temperature, these very cracks can become the source of a brittle fracture which is likely to propagate with the high speed characteristic of this phenomenon, and may lead to immediate failure. It is fortunate that the same measures of detailing, weld control, inspection, etc., which reduce danger of brittle fracture also tend to increase fatigue life. The possibility that a fatigue crack may "trigger" a brittle fracture emphasizes, however, that special precautions are advisable, particularly by using steels of low notch sensitivity in addition to careful detailing, when structures which are exposed to low service temperatures are also subject to significant stress cycling.

Plastic Design.—It is not the writer's intent to discuss in detail any of the phases of plastic design. A remark or two may be appropriate, however, to the extent that they fit the general tenor of this paper with its emphasis on the influence of materials' properties on structural performance.

There is no doubt that some degree of ductility is essential to the safety and proper functioning of any steel structure, no matter whether designed elastically, plastically, or by guess. Equally, there is no doubt that a larger amount of ductility is necessary for plastically designed structures to attain their planned carrying capacity. This is so because, as mentioned previously, there must be enough ductility at all locations of plastic hinges so that the hinges which form first can undergo, without fracture, those often very large amounts of local plastic rotation which will make it possible for the last hinge to form. No such demands on plastic deformation underlie elastic design. As has been pointed out, numerous tests of full-size frames under laboratory conditions have shown that the requisite amount of ductility appears to be present for the full plastic strength to develop. One possible reservation in regard to this feature was presented under the heading "Steel: Brittle Fracture." Another concerns the following:

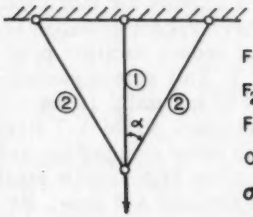
By far the majority of tests on plastic theory have been made on rolled shapes, or on structures fabricated from rolled shapes. For the method to be universal and applicable to structures of large size, its validity for fabricated structures may need further exploration. There has been recent evidence(12) that customary ways of preparing fabricated edges may reduce ductility practically to zero at temperatures as high as 40° F to 80° F. This refers particularly to sheared edges, to a somewhat lesser extent to manually flame-cut edges, and to arc strikes on edges. In this respect, rimmed ASTM A-7 steel proved considerably worse than semi-killed steel of the same designation, and structural silicon steel (ASTM A-94) as well as a low-alloy, high-tensile steel (ASTM A-242) proved to be about as unfavorable as the rimmed A-7 steel. Except for the semi-killed A-7 steel, brittle fractures in static tensile tests at the above, relatively high temperatures were obtained at stresses equal to and sometimes somewhat below the yield point of the material, particularly for sheared edges. It is clear that if, in fabricated construction, such edges were permitted to occur at yield-hinge locations, the attainment of the necessary plastic rotation prior to possible fracture would be in serious doubt. This is but one more illustration of the fact that the more design is based on actual strength rather than on nominal stress, the more important becomes information on structural performance of the material and development of adequate safeguards.

Even though it may be necessary to surround plastic design with more refined safeguards than are necessary for conventional procedures, there is no doubt that it represents impressive progress in three respects: (a) It enables the designer to utilize hitherto hidden plastic strength reserves, and thereby to design more economically. (b) It has furnished the designer and researcher with a deeper understanding of actual, as compared to idealized, structural performance. (c) It has added a new degree of freedom to structural design whose full possibilities will evolve only gradually with increasing usage. It is on this last feature that the writer wishes to dwell briefly.

When dealing with elastic continuous structures, the designer has only limited scope in shaping the structure for optimum performance and economy. A fixed-ended, uniformly loaded elastic beam of constant section has end moments

which are twice as large as the span moment, and hence most of the beam is understressed. If the designer wishes to improve the situation by reinforcing the ends he will find that, in the process, he has further increased the end moments at the expense of the span moment, in view of the added rigidity of the outer, reinforced portions. The stresses are likely to be as uneconomically distributed as before. In fact, it appears to be true that one cannot produce an elastic indeterminate structure of economically optimal stress distribution. This is illustrated by the simple, indeterminate system of Fig. 6. Economically optimal stress distribution would mean that the stresses in all three bars be equal. The elementary analysis in Fig. 6 shows that this is possible only for $\alpha = 0$, that is, when the indeterminate structure transforms into the determinate one which consists of one single bar. This example and that of the fixed beam illustrate the degree to which, for elastic conditions, the designer is hamstrung in his endeavor to optimize the shape of his structure.

In contrast, in plastic design, the designer is entirely free to dictate to the structure the manner in which he wishes it to perform. For the elementary model in Fig. 6, optimum plastic stress distribution results automatically since the structure will fail by unrestricted yielding only when the yield stress is attained in all three bars. This illustrates the economy of plastic design. In



ELASTIC	PLASTIC
From Deformation Compatibility	From Yield Condition
$F_2 = F_1 \frac{A_2}{A_1} \cos^2 \alpha$	$F_1 = A_1 \sigma_y$
$F_1 = A_1 \sigma_1$; $F_2 = A_2 \sigma_2$	$F_2 = A_2 \sigma_y$
Optimum Design With	$\sigma_1 = \sigma_2 = \sigma_y$
$\sigma_1 = \sigma_2$ Impossible	Automatic, Regardless
Except When $\alpha = 0$	Of α

FIG. 6.—"OPTIMUM DESIGN" OF INDETERMINATE STRUCTURES

addition, however, if for some reason the designer wishes to impose the condition that, at failure, the force in each of bars 2 be one-half of that of bar 1, all he has to do is to make the area of the former one-half of the area of the latter. Or, for the previous case of the fixed-ended beam, if he wants the support moments at failure to be one-half of the span moment (possibly because the restraining structures cannot resist more than this amount), all he has to do is to shape the beam so that the plastic section modulus at the supports is one-half that at the center. To state it generally, the plastic method enables the designer to dictate to the structure the precise manner in which he wants it to resist the external forces, to tell the structure how much of the total static moment he wants to be resisted at one section, and how much at another. It is this which makes the plastic method primarily one of design, while the elastic method with its limited freedom of action is primarily one of analysis.

It is likely that this new freedom of shaping structures optimally, in regard to economy in some cases, in regard to aesthetics in others, will be appreciated only gradually as designers, through use, become aware of the new potentialities of this approach.

CONCRETE AND REINFORCED CONCRETE

In comparison with steel, concrete is a much more complex and less predictable material. This is due to two entirely independent causes. From the fundamental viewpoint of materials behavior, the high degree of heterogeneity of concrete, the difference in basic properties of its various constituents (coarse and fine aggregate and hardened cement paste), the pronounced effects of creep and shrinkage, and the fact that the tensile strength is a small fraction of the compressive strength, represent some of the causes why the understanding and prediction of performance of concrete under load has not attained the same degree of development as for steel and for other ductile materials. From the practical viewpoint the fact that concrete on the whole is produced under less controlled conditions and that its final properties depend to a greater degree on job operations than is true for steel (placing, curing, etc.) adds to the lesser degree of predictability which still prevails in regard to many features of concrete behavior. (It should be noted, though, that recent investigations into the true effective properties of steel, as discussed previously in regard to yield strength, residual stress, brittle fracture, etc., have shown that these properties, too, exhibit greater scattering and dependence on fabrication operations than had previously been suspected.)

The difficulties are compounded in reinforced concrete by the complex mode of interaction of the two constituent materials and by the variety of ways in which they can be combined to form structural members. The fact that our understanding is still lagging in regard to some of the basic features of reinforced concrete, such as its behavior in shear and diagonal tension, in torsion and combined torsion and bending, its long-time deflections under load, etc. is by no means amazing. On the contrary, it should be a matter of some pride that, in spite of these fundamental complexities, such features as the strength in compression, in bending, and in combinations of compressing and bending, have been brought under very satisfactory control while in regard to the enumerated, more complex, features, methods have established themselves which, though admittedly crude and lacking of rigour, have proven themselves as adequate design tools.

Cylinder Strength.—The single, most important structural property of concrete is its cylinder strength. Wherever, in reinforced concrete, the carrying capacity is governed by the concrete rather than the steel, it is strongly dependent on, and in most cases directly proportional to, cylinder strength. In addition, and in contrast to steel, the deformation characteristics of concrete structures (modulus of elasticity, etc.) are also very largely dependent on cylinder strength. The degree to which actual cylinder strength in the finished structure corresponds to the value on which the design has been based is, therefore, of the utmost importance.

Figs. 7(a) and (b), represent, respectively, 861 tests of 33,300 cu yd and 296 tests of 3300 cu yd, exhibit the influence of good versus poor concrete quality control.(5) For the concrete of Fig. 7(a) the desired minimum strength was 3.8 ksi, for Fig. 7(b) 4.3 ksi. while the actual mean strengths were 5.2 ksi and 5.8 ksi., respectively, a spread of 35 to 40%. For the well-controlled concretes of Fig. 7(a) the percentage of cylinders which tested below the design strength of 3.8 ksi is a fraction of a percentage. In contrast, more than 15% of the cylinders of Fig. 7(b) failed to meet the stipulated minimum strength in spite of the wide spread between the actual mean and the stipulated minimum strength,

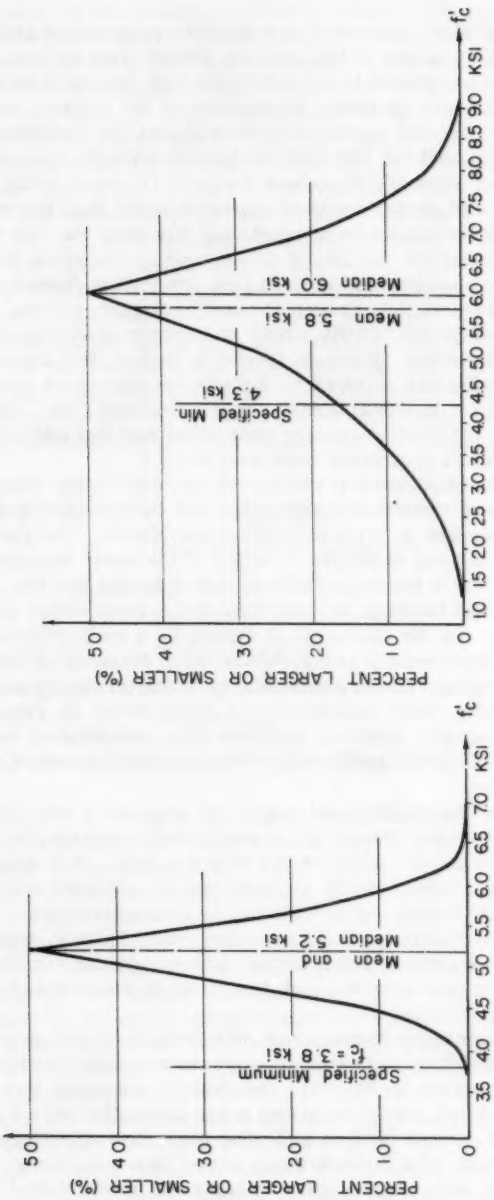


FIG. 7.—PROBABILITY CURVES FOR CONCRETE CYLINDER STRENGTH

and some of the cylinders tested as low as one-half of the desired value. These statistical data emphasize strongly that only the combination of careful concrete control with a sufficiently wide spread between the desired minimum strength and the mean strength for which the mix is designed will assure adequate quality. Some codes require that the water-cement ratio be so chosen as to result in a strength 15% above the desired design strength. In view of the foregoing information there may be some doubt whether this spread is sufficient.

Tensile Strength.—Since reinforced concrete structures are so arranged that the strength of concrete in direct tension is rarely of practical significance, not much attention has been paid to this property. A further reason for this neglect may be found in the experimental difficulty of determining concrete tensile strength by test.

However, the strength under combined stress of a material like concrete, such as in shear, in torsion, or in a combination of these with bending, can be analyzed only if, in addition to the compressive strength, at least the tensile strength is known. This is immediately evident from Mohr's strength theory which, while not necessarily the most exact, is probably the simplest of the various strength theories applicable to a material of the nature of concrete. Lack of information on this property, therefore, seriously hampers attempts toward a deeper understanding of concrete performance.

The statement can often be found that the tensile strength is some reasonably constant fraction, such as 1/10, of the cylinder strength. It has recently been maintained[13] that this ratio can vary within the wide limits of 1/8 to 1/30 and that its most frequent range is between 1/10 to 1/16. It seems likely, therefore, that the tensile strength is a separate property of any given concrete and is not uniquely defined by some definite relation to the cylinder strength. The possible practical importance of this situation will be discussed subsequently in connection with combined stress.

Stress-Strain Curve.—Until recently, strain measurements during cylinder tests have represented the primary source of information on stress-strain curves of concrete. Such tests give adequate data at relatively low stresses (within the working range). There was considerable doubt, however, whether these data were representative of actual concrete behavior at near-ultimate stresses, that is, in the region which matters most for strength determinations. The doubts were two-fold: (a) It was suggested by C. S. Whitney that the sudden failure of test cylinders may frequently be precipitated by the elastic properties of the testing machine, rather than those of the material; (b) it was dubious to what extent the shape of the curve obtained from cylinder tests was representative of the stress distribution in the compression zone of a member subject to bending or eccentric compression. Both these questions are of decisive importance for a rigorous underpinning of ultimate strength design.

In very recent years, independently and almost simultaneously, satisfactory light was shed on these questions by evidence obtained by ingenious test methods in Germany[14] and in the United States.[15] These investigations showed that: (1) In cylinder tests the stress-strain curve upon reaching ultimate strength is indeed cut short by the properties of the testing machine, particularly for higher strength concretes. (2) The distribution of concrete stresses over the compression zone of flexural members is, in fact, closely similar to the stress-strain curve in concentric compression. (3) Most important, the shapes of the stress-strain curves of concretes of various strengths are of the character shown on Fig. 8.[15]

It is seen that concrete behaves reasonably elastically in the range of moderate stresses. There is, however, marked departure from linearity at near-ultimate stresses. In this respect concrete behaves quite differently from decidedly brittle materials, such as cast iron or glass, and can be said to possess a certain amount of ductility. This ductility, however, is seen to decrease with increasing cylinder strength. There is a certain paradox in the fact that recent tendencies to use ever higher concrete strengths coincide with simultaneous tendencies toward utilization of the ductility of concrete in ultimate strength design. It is probable that the tendency toward the very high concrete strengths is caused by the fallacious premium which the straight-line theory places on the use of such concretes in flexural members. With broader acceptance of ultimate strength design a more realistic attitude toward concrete strength is likely to establish itself.

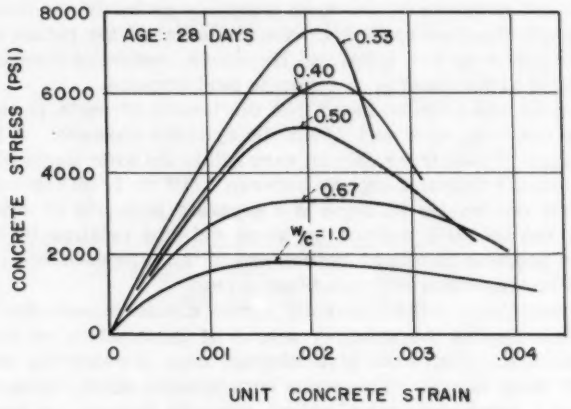


FIG. 8.—STRESS-STRAIN CURVES FOR CONCRETE

A very remarkable feature of the stress-strain curve of concrete is that it shows a descending branch after the maximum stress has been reached. The fact that its shape, in this respect, is not very different from that of certain ductile metals is deceptive. In the latter, such as in aluminum alloys, steel, etc., the descending branch pertains only to the nominal stress, referred to the original area. Since, in this region of large, ductile strains, the reduction of area is considerable, a plot of true stress versus strain invariably shows an ascending curve right up to fracture. In concrete, in contrast, in view of the relatively very small strains, no significant change of area occurs with increasing stress so that the conventional stress-strain curve differs very little from that of true stress. It seems that no other structural material possesses this descending branch, with the possible exception of timber under certain conditions. It is suspected[15,16] that this branch is caused by progressive, irreversible microcracking so that, with further increasing strain, a progressively smaller number of uncracked particles is called upon to resist the load.

The total duration of the loading process for each of the tests performed by E. Hognestad, et al[15] was about 15 min. This demonstrates that the shape of these curves is not a consequence of creep, as is sometimes stated, at least not within the normal meaning of the term creep, but represents correctly the performance of concrete at reasonably high rates of loading.

An important feature of these curves is the fact that their peaks occur, that is, the maximum stresses are reached, at strains of the order of 0.0020 to 0.0025, for all tested mix proportions and cylinder strengths. Column design has long been based on the assumption, proved in many tests, that the strength of a short, concentric column is simply the sum of the yield strength of the reinforcement plus the crushing strength of the concrete. This situation is evidently possible only if the concrete will not crush or substantially enter the descending branch of the curve before the steel has reached its yield point. Since, in steel, an elastic strain of 0.001 corresponds to a stress of 30 ksi, it is seen that column reinforcement with yield points up to 60 ksi to 75 ksi will indeed reach these yield points before the concrete starts weakening, as is assumed in present column formulas. This may not be true, however, for steels of significantly higher yield strength. The same situation may hold for compression reinforcement in flexural members. This may put a limit on the efficiency of steels of very high strength for compression reinforcement.

The descending branch of the stress-strain curve comes into play primarily in beams with very high reinforcement ratios, when strength is governed by crushing of the concrete. While such high ratios are rare, the possibility that in this range the integrity of the concrete is impaired by microcracking remains a somewhat disturbing feature. It could conceivably affect structural safety in the case of repeated applications of near-ultimate loads, and may warrant additional investigation.

Flexural Strength of Reinforced Concrete.—Flexural failure ultimately consists in the crushing of the concrete compression zone. If this crushing is preceded by yielding of the steel (for low steel ratios), then the ultimate moment at which crushing occurs differs very little from that at which yielding begins. For this reason the ultimate moment of beams which fail in tension can be computed with high accuracy, provided the reinforcing steel has a horizontal yield plateau.

If crushing takes place while the steel stress is still below the yield point (high steel ratios), the ultimate moment cannot be predicted with any comparable precision. The best that can be done is to calculate a reliable lower limit (see Fig. 4, Ref. 17). It has been proposed that for this situation the maximum concrete strain at the outer compression fiber be taken as a failure criterion. It is seen from Fig. 9[18] that ultimate strength cannot be computed from a maximum strain criterion. That is, there seems to be little prospect in establishing that concrete will crush at a certain, predictable magnitude of the strain at the extreme fiber since tests have failed to indicate reasonable correlation between strain and strength. Fortunately, with standard-strength reinforcement, the steel ratios at which strength is governed by concrete crushing prior to steel yielding are so high as to be rarely of interest. This is not so when high-strength reinforcement is used. In addition, certain high-strength steels which are coming into use both in Europe and, lately, in the United States, do not have a flat yield plateau but enter strain hardening immediately upon reaching the yield strength. In that case, even for low steel ratios, the flexural strength cannot be computed by the usual methods of ultimate strength theory and the only statement that can be made is that the strength of such beams

is larger (by up to 40%) than computed by assuming the steel to possess a flat yield plateau.(19)

It is seen, then, that in view of the complex nature of concrete, the strength of flexural reinforced concrete members so far is calculable with precision only for the simplest of situations (under-reinforced beams with flat-yielding steel). It is fortunate that this is, by far, the most frequent situation, and it is equally fortunate that for all other situations, if not the strength itself, then at least a reliable lower limit can be easily computed. Investigations are under way both in the United States and in Germany which attempt to develop a basically different approach to ultimate flexural strength, based on the concept that internal stresses readjust themselves to maximize the resisting moment. In the meantime, however, there are arguments in favor of limiting the maximum admissible strain even though there appears to be no reliable correlation between it and flexural strength. If that strain is limited, for low and medium strength concretes, to 0.003 or possibly to 0.0025, and to somewhat smaller

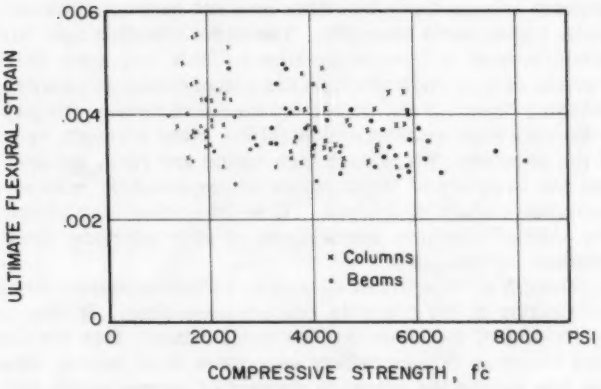


FIG. 9.—ULTIMATE STRENGTH AT FAILURE OF REINFORCED CONCRETE MEMBERS

values when f'_c exceeds about 4500 psi (Fig. 8), one would thereby eliminate from utilization the descending branch of the stress-strain curve. If this branch were actually caused by irreversible microcracking, this may be a desirable measure.

Failure under Combined Stresses.—The difficulties cited in predicting strength for the essentially uniaxial conditions of flexural compression are compounded in the case of combined stress, that is, of stresses acting in two or three directions. The shear strength of beams and the strength of members in combined bending and torsion (with and without transverse shear) are but two examples of such biaxial, combined stress. They emphasize the importance of developing a method for predicting failure under these conditions, and several promising attempts have been made on this problem in recent years. The matter must be looked at in two different ways: For one, it is a problem of basic, pure research in which an answer is sought for which only one criterion exists, namely that it be reliable, regardless of possible complexity. The other is the fact that in order to be of engineering use, the infor-

mation obtained in this fundamental manner must be capable of sufficient simplification to permit use in routine design. In both these respects the problem of concrete failure under combined stress is still in its initial stages.

Of the various, customary strength theories, the simplest which could be applied to a material of the type of concrete is that of Coulomb (identical with Mohr's theory with straight envelopes). It regards failure as a sliding on inclined planes and postulates that the resistance to such sliding consists of two parts: the internal cohesion, and the sliding friction. The latter, in turn, depends on the coefficient of internal friction and on the normal stress on the plane of sliding. H. J. Cowan[20] maintains that two types of concrete failure must be distinguished. He suggests that crushing failure is governed by Coulomb's theory, but that cleavage failure (tension cracking) is directly caused by the maximum principal tension stress (Rankine theory). On this basis he shows reasonable agreement with a moderate number of tests on beams in combined bending and torsion. While it is known that the Coulomb-Mohr theory is fundamentally deficient in that it neglects the influence of the intermediate, principal stress, this deficiency is rarely important in ordinary type members where stresses are usually biaxial rather than triaxial.

D. McHenry and J. Karni[21] find that the Mohr theory does not correlate well with their tests of tubes subject to longitudinal compression and circumferential tension. They attempt correlation with octahedral shear and normal stresses, but conclude:

> "At the present stage of our knowledge the accumulation of test data is perhaps more important than is the search for simplified criteria. The present data indicate that the relationship between the principal stresses for tension-compression failure is rather complex and is not well defined."

At the same time B. Bresler and K. S. Pister[22] observe, very much to the point

> "that strength of concrete is a function of the state of stress and can not be predicted by limitations of tensile, compressive and shearing stresses independently of each other".

It is evident, therefore that "the search for simplified criteria" is of great practical importance, since without them design for shear, torsion, and other combinations must remain on the unreliable basis of considering "the stresses independently of each other." In analyzing the results of their tests of tubes submitted to compression and shear (torsion), Bresler and Pister attempt to relate their results to the octahedral normal stress and to a slight modification of the octahedral shear stress, and they extend their method to the problem of the shear strength of beams without web reinforcement, under strictly specified, restrictive conditions.

These various approaches have in common the fact that they attempt to relate the strength of concrete under combined stress to a single property of the material, usually the cylinder strength or some modification thereof. In a doctoral dissertation prepared under the writer's supervision, S. A. Guralnick[13] maintains that the strength of concrete under combined stress can be understood only if at least two properties are known, the tensile as well as the compressive strength, and that these two properties are largely independent of each other. This has been discussed previously. To illustrate: Cowan's method is based on the assumption that the ratio of tension to compression

strength is about 1 to 9 while Bresler and Pister's criterion would result in a tensile strength of $f'_c/13.2$; at the same time even the limited test evidence by McHenry and Karni shows a spread of measured tensile strength from $f'_c/11$ to $f'_c/17$. It stands to reason that strength hypotheses which do not account for this variability are not likely to be reliable. Guralnick utilizes the Coulomb-Mohr approach to derive relatively simple interaction-type criteria and applies them to results of only such tests for which the compressive and tensile strength of the particular concrete was known by independent tests. The reasonable correlation which he obtains may be regarded as one further contribution to the urgent problem of combined strength of concrete, whose satisfactory solution is not yet at hand.

Shear Strength of Beams.—The most important type of failure in concrete structures which is caused by combined stress is that usually designated as "shear failure." This occurrence is, generally, of a "brittle" character, that is, it takes place without warning and leads to immediate and total collapse. This is in contrast to the gradual way in which under-reinforced beams fail in flexure, or spiral-reinforced columns in compression. It is the reason why it is advocated that in ultimate strength design a larger margin be provided against shear than against flexural failure, so that structures so designed would give due warning of distress.

An unusually large amount of research on shear strength, within the past 10 yr, has to a considerable extent clarified the mechanism of shear failure in a qualitative manner. At the same time it seems that none of the quantitative conclusions, some of them in the form of very elaborate equations, possess sufficient generality to be applicable, with confidence, outside the range of variables of the tests from which they were derived. One of the reasons for this, apart from the basic complexity of shear strength, particularly in the presence of web reinforcement, is probably the fact that no attempt was made by these investigators to analyze shear failure as involving strength under combined stress. The necessity for this latter approach was pointed out by P. M. Ferguson.(23)

Shear failure in beams without web reinforcement proceeds in two stages: First, a "diagonal tension crack" forms, caused by the inclined tension and compression stresses which prevail in the uncracked section. The correct determination of that state of stress, and of that shear force which will cause such a crack to form is, therefore, clearly a matter of combined stress. Once such a crack is formed, the external shear is resisted only by the remaining uncracked concrete on top of the tip of the crack. Final failure occurs when this concrete is destroyed by the combined action of the flexural compression and the shear (Fig. 10(a)). This, evidently, is again a matter of failure of concrete under combined stress.

In beams with web reinforcement an additional stage is interposed between these two. That is, after a diagonal crack has formed, resistance is provided jointly by the stirrups crossed by the crack and the remaining, uncracked compression concrete (Fig. 10(b)). When the stirrups have reached their yield point, the effects of any additional load increase must be resisted by the remaining compression concrete, so that final failure, again, is caused by combined stress in this part of the beam. The foregoing represents a possibly oversimplified description of the mechanism of shear failure.

The writer knows of only two attempts to apply conditions of strength under combined stress to the problem of shear in beams.(13,22) Both these attempts show reasonable correlation with a limited number of beam tests. Neither of

them pretends to have furnished more than the beginnings of a basic solution of the problem of shear strength. There is a school of thought which maintains that, in view of the brittle character of shear failure, ultimate strength design for shear should not be based on actual collapse but on the immediately preceeding stage, described previously. That is, for beams without web reinforcement, design would be based on that load which causes the first diagonal crack, and for beams with web reinforcement, on that load which causes the web reinforcement to yield. It is clear that this more conservative approach, likewise, cannot avoid the nature of the problem as one of combined stress.

If this is the state of our knowledge on shear strength (as of 1959), it might be asked how it is possible that we have been designing, for these many years, concrete structures which appear to be reasonably safe in shear. The answer is that our present shear-design procedures are based on certain simplified concepts in combination with test evidence and experience. They result in safety factors which range widely between the limits of about 2.5 to 7 and more.

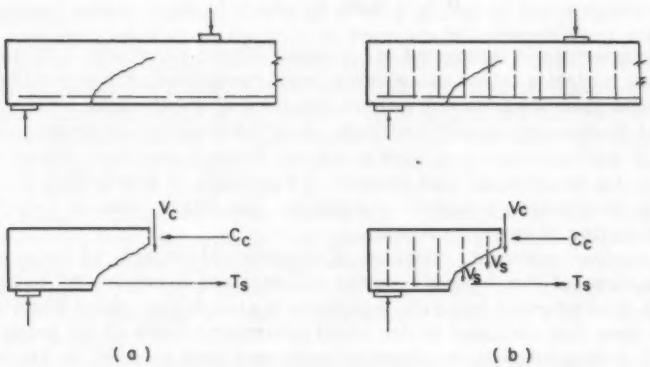


FIG. 10.—"SHEAR FAILURE" MECHANISM OF REINFORCED CONCRETE BEAMS (SCHEMATIC)

Occasionally safety factors have fallen sharply below these limits and have resulted in actual structural distress or failures which were traced to shear weakness. This means that present procedures are normally adequate, but by a margin which is rarely known with any reasonable accuracy. Correspondingly, shear designs in many cases are uneconomical while others, fortunately in a much smaller number of cases, are not adequately safe.

Time Effects.—Under normal temperatures, steel is insensitive to loads of long duration so that the only time effects of importance to the structural designer are those connected with high strain or load rates. In contrast, concrete under stress continues to deform or creep for a long time at a decreasing rate; it stabilizes only after months or years. Considerable attention has been paid recently to creep in view of its importance in the design of dams, arches, and prestressed concrete. Based on extensive experimental evidence, a number of empirical formulas have been developed which describe the observed relationships between creep, stress, and time.[24]

Since concrete is subject to creep and reinforcing steel is not, it is evident that sustained loading causes a redistribution of internal stresses in reinforced concrete members. In fact, when the first extensive evidence was obtained, in the 1930's, that the stresses and the strength of columns do not conform to elastic relationships, it was thought by many that this was due to stress redistribution caused by creep. It is evident that this is not so. For one thing, the duration of most column tests from which this evidence was obtained was one of hours or fractions thereof, rather than months. For another, the concrete stress-strain curves of Fig. 8 with their pronounced inelastic portions were obtained at a steady strain rate which resulted in failure within the short time of about 15 min. Possibly the most spectacular evidence to the effect that plastic or inelastic action of concrete is not dependent on creep is presented by some column tests.[25] Ten columns of different types were tested to failure within less than 6 min each, and their ultimate loads compared with the basic formula for inelastic or plastic column strength,

$$P_{ult} = 0.85\, f'_c\, A_c + f_y\, A_s \quad \ldots\ldots\ldots\ldots\ldots \quad (5)$$

The average ratio of measured to computed ultimate load was 1.02, which indicates that inelastic action establishes itself completely, for strength purposes, within less time than the very short duration of these tests. (The writer is aware of the theoretical difficulties in clearly distinguishing between creep and plastic or inelastic action at high stresses. From a practical design viewpoint he would like to draw the line between the duration of short-time live loads in ordinary structures, a matter of minutes, and either dead or long-time live loads, a matter of months or years.)

The reverse question, which is of considerable practical consequence, is this: In view of the internal stress redistribution caused by creep, is the strength of reinforced concrete members in structures under sustained loading less than that obtained in the usual laboratory tests under relatively fast loading? In this respect two types of sustained loading must be distinguished. (1) A member or structure carries, for a long period, the full design load and is then loaded to failure at a reasonably rapid rate. This is the case of primary practical interest for the normal structure which is essentially used and loaded as designed, but which may be subject to short-time catastrophic overloads, such as by earthquakes, hurricanes, dropping loads, or the like. (2) A member or structure carries near-ultimate loads indefinitely or until these high, sustained loads cause collapse. In actual structures this case would be of interest only when, in design, long-time loads have been underestimated by a factor approaching 2, or when a structure is irresponsibly misused, such as the use of a former office building for heavy warehousing.

Test evidence shows[26] that columns subject to sustained loads of type 1, above, develop the same strength as columns of the same age failed under short-time loading. In regard to the much severer sustained loading of type 2, tests indicate for concentric[27] and for eccentric[28] columns that the sustained-load strength is rarely less than about 90% of the short-time capacity. In regard to beams, evidence is not too ample. Those tests which have been made[29,30] seem to indicate that for beams without and with compression reinforcement the long-time strength under the severer sustained loading of type 2 is indistinguishable from the short-loading strength of identical members of the same age. It seems, therefore, that creep does not present a significant problem in reducing strength under sustained load and that, consequent-

ly, design can continue to be based on the ample test evidence of the short-time strength of reinforced concrete members.

That effect of creep, therefore, which is of primary practical importance is the long-continued increase in deflection under constant, sustained load. These additional deflections can be sizeable, particularly in shallow members, where they can amount to two and more times the value of the instantaneous deflections obtained when loads are first applied.(29,31) Present tendencies are toward the use of shallower members, facilitated by the introduction of ultimate strength design and of high-strength reinforcing steels. In this connection it becomes increasingly important to develop design tools for preventing long-time deflections from becoming excessive. It appears that accurate methods for predicting long-time deflections have not yet been developed. For the time being, reasonable estimates can be obtained if the moment of inertia of the transformed cracked section is used, using for the modulus of elasticity of the concrete one-third to one-fourth of the customary value for short-time loading.

At the opposite end of the range of time effects is that of the properties of concrete at very high rates of strain, that is, when loads increase very rapidly from low values to their maximum or when the structure is subject to impact. Such evidence as exists(32) shows, fortunately, that crushing strength increases significantly with increasing strain rates (up to 80% in the reported tests), that the stress-strain curve in dynamic loading shows the same development of an inelastic range prior to failure as in static loading, and that the energy absorption capacity at high strain rates is certainly not lower, and frequently considerably higher than under static conditions. It seems to follow from this that if members are designed with full utilization of the inelastic properties of concrete (ultimate strength design), they will not fail under high strain rates at loads which are lower than those calculated by ultimate strength theory, even though the test evidence on which this theory is based was obtained at relatively slow loading.

Another time effect, to whose investigation and control too little attention has been paid recently, is shrinkage. In connection with the introduction of higher strength steels and other new developments (limit design, etc.) much effort has been and is being spent on investigating cracking of concrete as it is caused by tension stresses in flexure, shear, and combinations thereof. It so happens that these stress cracks are almost the only type that will occur in laboratory specimens of the usual, unrestrained kind. On the other hand, observation of existing structures shows, as a rule, that much more extensive and obnoxious cracking is caused by shrinkage than by stress. In addition to cracking, shrinkage also can cause warping and deflection of unsymmetrically reinforced members (since shrinkage will be resisted where steel is located, such as on the tension side of beams, but will proceed relatively unhindered on the compression side unless compression reinforcement is used). These deformations can attain sizeable and objectionable proportions, particularly in shallow members. Finally, sizeable internal tension forces due to shrinkage can develop in structures if adequate provisions for relieving restraint (shrinkage joints which work) are not made. These stresses have been known to add to those caused by loads in a manner which can result in serious distress and even failure.

Ultimate Strength Design and Limit Design.—Ultimate strength design is now an accepted procedure in several parts of the world and, since 1956, has been a permissible alternate method in the United States. The term has come

to mean the utilization in design of the ultimate strength of individual reinforced concrete sections, as distinct from the load capacity of entire structures composed of members having such sections. This method is based on the more complete utilization of the actual properties of both steel and concrete, in contrast to the idealized elastic behavior on which the conventional straight-line theory is based. Historically, ultimate strength design developed rather in the inverse order, that is, phenomenologically. Tests of reinforced concrete members (beams, columns, etc.) showed that they did not behave as predicted by straight-line theory. From this it was evident that inelastic properties, though not fully known, must influence strength decisively. Methods were developed which would more closely reflect actual behavior of structural members based primarily on tests of members and then on some rather crude working hypotheses in regard to inelastic materials properties. Only recently has considerable light been shed on this basic materials performance, and refinements are now being made in ultimate strength design based on this better understanding of the fundamentals.

In addition to the actual strength of sections it has been found that the inelastic behavior of both steel and concrete leads to a redistribution of moments at high loads in indeterminate reinforced concrete structures in a manner analogous to that utilized in plastic design of steel structures. The corresponding design method of reinforced concrete structures is coming to be known as limit design.(33) The differences between plastic design of steel structures and the developing methods of limit design of reinforced concrete structures are largely a consequence of the different performance under load of these two materials. Thus, the usable plastic rotation capacity of rolled-steel shapes usually far exceeds that required for the development of adequate plastic hinges (questions of local buckling being beyond the scope of this paper). In reinforced concrete the moment-curvature relationship, beyond the limit of elastic action, is non-linear, similarly as in steel. However, in contrast to hinges in steel, undesirable consequences of various kinds may accompany plastic rotations in reinforced concrete unless adequate safeguards are provided.

For members with low steel ratios such plastic rotation is primarily caused by yielding of the reinforcement. This yielding, however, is likely to be accompanied by large-scale tension cracking. If such cracking occurs only at high (near-ultimate) loads it may be harmless in regard to serviceability of the structure. Yet, if plastic rotation leads to excessive cracking at service loads (at or slightly in excess of the design load) it may be thoroughly objectionable. From this viewpoint it is deplorable that in many valuable investigations on moment redistribution little detailed information has been obtained, or published, on crack formation.

On the other hand, for members with medium and high steel ratios, plastic rotation is necessarily accompanied by relatively large concrete strains. To what extent such large strains are admissible is still a matter of some dispute, even as far as a single loading to failure is concerned. Previously the possibility has been mentioned herein that part of the inelastic strain of concrete at high stress may be caused by irreversible microcracking. If such microcracking develops at plastic hinges, the question of repeated applications of near-ultimate loads assumes more importance in regard to structures designed by limit-design procedures than it has in structures whose moments and shears have been calculated by elastic analysis. In this respect one might conceivably speak of a shake-down problem in reinforced concrete structures in a different sense from that in steel structures. In the latter, shake-down implies the

consecutive application of different load patterns, in which case the structure may or may not "shake-down" to a stable condition (in most cases it does). In reinforced concrete a shake-down problem may exist not only for successive application of different load patterns but even for repeated applications of the same loading.

These possibilities are mentioned not in order to cast doubt in advance on the possible advantages and merits of limit design in reinforced concrete. They are discussed, merely, to indicate that, in view of the greater complexity of the pertinent materials, it is likely that limit design in reinforced concrete will take different forms and will have to be surrounded with more elaborate safeguards than plastic design in steel.

Limit design, therefore, represents a prime example of the fact that thorough knowledge of the performance of materials under load, in all its various aspects, is the chief prerequisite for improving design methods. It has been the aim of this broad review to facilitate acquaintance with the present state of our knowledge in this field, so that newly developing design methods can be utilized with understanding and discretion.

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GOLDEN GATE BRIDGE VIBRATION STUDIES[a]

Closure by George S. Vincent

GEORGE S. VINCENT,[1] F. ASCE.—The writer appreciates the closely reasoned discussion by Mr. Balog. He concurs in the view that it is very desirable to have wind observations at several points on a bridge in order to correlate the bridge behavior with the wind data. He would go further than this and include also measurements of the vertical angularity of the wind which has been found to be significant. It is a mistake, however, to imply as Mr. Balog appears to have done, that the limitations described concerning Mr. Cone's impromptu observations on the bridge with no means for making measurements or correlating simultaneous observations apply also to the recorded behavior of the bridge and the wind velocity over the period of years of the tests. These latter observations were rather complete as to a motion of the bridge and were correlated in detail with the wind measurements at the middle of the bridge. In the case of the Golden Gate Bridge the wind does not vary excessively over the length of the structure. If it did, the consistency and correlation shown by Figs. 12 and 15 would not have occurred.

Mr. Balog states that "the vibrations of the Second Tacoma Bridge and more recently the Mackinac Straits Bridge have also been measured." The writer is not aware of any such measurements. In the case of the Tacoma Bridge he does know that inspection of the structure during high wind soon after it was built showed no appreciable movements of the structure although there was minor oscillation of individual suspenders. In the absence of any evidence of wind-excited oscillation administrative authorities are not justified in going to the expense of providing the necessary instruments for recording motions adequately. It must be borne at mind that improvised observations made on the spur of the moment when a storm occurs are completely inadequate, particularly in view of the fact that many such storms occur in hours of darkness. It is doubtful that any justification has been found for installing equipment for measuring motions on the Mackinac Straits Bridge.

The effect of adding bottom laterals to the Golden Gate Bridge was not included in the research covered by this report and for this reason, that feature was discussed only briefly in a footnote. However, the data which Mr. Balog has requested had been previously published. Vol. 150, No. 4 of Engineering News Record for Jan. 22, 1953, contains an article on page 22 which gives the desired information concerning the new lateral systems. Briefly, the minimum member consisted of four - 4 in. by 4 in. by 7/16 in. angles laced in a box section. The change added 1400 lbs per lin ft to the dead load of the bridge.

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[a] October, 1958, by George S. Vincent.

[1] Chf., Bridge Research Branch, Bur. of Pub. Rds., U. S. Dept. of Commerce, Washington, D. C.

The significant effect of the addition of the bottom lateral system was to increase the frequency of vibration of the structure in its symmetric and asymmetric torsional modes. It had essentially no effect upon any vertical oscillation as is noted in the concluding statement of the footnote of the original paper.

Mr. Balog refers rather vaguely to the indications of experience concerning the desired rigidity of a suspension bridge and speaks of drawing inferences concerning required torsional bridge rigidity, of suspended structures from the observations on this one bridge, the Golden Gate Bridge. The profession can do better than this. The effect of bottom laterals or other structural elements upon the natural frequency of oscillation of the structure can be computed. Section Model tests on the particular structure—not some other bridge—will show the frequency necessary to avoid oscillation in a wind of a given velocity. It should be pointed out again that the frequency increases as the square root of the stiffness.

The reference to Menai Straits Bridge and other small structures is not entirely applicable to our thinking on large bridges. It is readily possible to increase both the vertical and torsional stiffness of a small structure by means of the stiffening elements. This is possible to a much less degree in the case of large structures.

The writers reasoning was misinterpreted if it was thought that he implied that the primary obstacle in the way of building up higher amplitudes in the first asymmetric mode was the lifting action at mid-span. This was analyzed in an attempt to explain the somewhat increased frequency observed in the torsional mode as compared to that computed by the usual assumptions. Undoubtedly this action contributes some to damping, both through frictional and dynamic effects as described in the paper. However, the principal factor, discussed as item (1), was the fact that the higher velocities were of too brief duration to build up the amplitudes which a steady wind of such velocity could have excited.

ELASTIC RESISTANCE OF REINFORCED CONCRETE BEAMS[a]

Closure by G. R. Swihart, J. R. Allgood, and W. A. Shaw

G. R. SWIHART,[1] M. ASCE, J. R. ALLGOOD,[2] and W. A. SHAW,[3] M. ASCE.— The writers appreciate the views expressed in the discussion by Mr. Mavis. It is regretable that the title of the paper is not descriptive of the content. The paper as originally prepared was entitled, "Static Resistance of Reinforced Concrete Beams." However, the word "Static" was unexplainably changed to "Elastic" prior to publication.

The theory presented was compared with considerably more data than that indicated by Fig. 10 thru 13. Available data from the literature in which loads and deflections were recorded to ultimate failure, including the exhaustive investigation of reference 3 in the bibliography, were checked by the theory presented and found to be in reasonable agreement. The theory was thus exposed to a severe test in that the range of the variables was very large. Only those tests conducted at the U. S. Naval Civil Engineering Laboratory were included in the paper in order to conserve space, since the results were typical of the numerous comparisons made.

The "algebraic" approach is believed to be reasonable in presenting any general theory. The general equations, as such, admittedly appear to be rather complex. Solution of the equations, however, is very rapid when used in conjunction with the curves of Figs. 5 and 6. The "algebraic" equations were developed for use in a recently completed study of the behavior of reinforced concrete beams subjected to a dynamic load. Ninety design charts have been prepared, which enable the rapid selection of an "optimum" beam for resisting a given blast load. These charts will be published in the near future.

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[a] January, 1959, by G. R. Swihart, J. R. Allgood, and W. A. Shaw.

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CORRELATION OF PREDICTED AND OBSERVED SUSPENSION BRIDGE BEHAVIOR[a]

Closure by George S. Vincent

GEORGE S. VINCENT,[1] F. ASCE.—Mr. Farquharson's review of the trend in design of suspension bridges leading to more and more flexible suspended structures includes the names of four girder-stiffened bridges which were built in the late 1930's which culminated this trend. As he points out, they were designed for the static but not the dynamic effects of the wind.

It is the writers view that the designers of these four bridges should not be censured for following this trend and for neglecting dynamic effects. It remained for the Tacoma Narrows Bridge Research to reveal the mechanism whereby the Menai Straits Bridge was seriously damaged a century earlier. Moreover, a period of 80 yrs, during which successively larger and larger suspension bridges were built, elapsed between the last significant failure of a suspension bridge under wind action and the design and construction of these four referred to. Designers and the profession at large may be pardoned for neglecting dynamic wind effects in the face of experience tending to show that such difficulties were to be encountered only on the smaller bridges and had been overcome in modern practice.

Mr. Farquharson does well to point out that roadway slots are not uniformly beneficial and that details of the form of a structure modify materially the effect of the wind. It is these variations which counsel the aerodynamic testing of designs for proposed bridges of any importance at the present time, pending the accumulation of a body of specific data on the subject.

The variation in wind action on a given bridge, as illustrated by Mr. Farquharson's Fig. 3, suggests why many bridges in service are stable, that is, they have not encountered the wind conditions which could excite oscillation. To a degree, one may rely upon such variations in the design of proposed bridges but it may not always be wise to presume such a favorable action.

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[a] February, 1959, by George S. Vincent.

[1] Chf., Bridge Research Branch, Bur. of Pub. Rds., U. S. Dept. of Commerce, Washington, D. C.

DESIGN TABLE FOR STEEL COLUMNS WITH ECCENTRIC LOADS[a]

Closure by Richard Hugh Bigelow

RICHARD HUGH BIGELOW,[1] A.M. ASCE.—To further compare Mr. Levy's method and the writer's method consider the problem below:

Select the lightest 12-in. column section for a buckling length of 30 ft with a 310 kip load acting on the Y-Y axis 6 in. from the X-X axis.

Mr. Levy's method

$$F_A = \frac{193}{65} \times 3.4 = 10.1 \text{ ksi}$$

$$\frac{10.1}{20} \times 0.216 \times 6 = 0.654$$

Select column to carry 310(1.654) = 513 kips which corresponds to a 12 WF 161. Mr. Levy states no further calculations are necessary unless the initial selection of Bx and F_A were appreciably in error. The value of Bx for this section is 0.213 and does differ from the initial selection. However, it is smaller than the initial selection and would lead the designer to conclude that his answer may be conservative.

$$F_A = \frac{515}{161} \times 3.4 = 10.9 \text{ KSI}$$

This value is greater than the initial selection and would contribute to an increase in the equivalent concentric load. The designer at this point would check the 12 WF 161 and finding it over-stressed would select the 12 WF 190. The writer's method:

$$P_a \text{ (equivalent)} = 310 + (310 \times 6)\ 0.1172 = 528 \text{ kips}$$

The next larger value for P_a in the 12-in. series is the 12 WF 190.

The above problem is not typical. However, there are cases where the equivalent concentric load as calculated by Mr. Levy's method will fall between values in the column tables and neither of the corresponding sections will suffice. This will necessitate further calculations. With the writer's method the larger section will always suffice and occasionally the lighter section will suffice. It will be left to the reader's judgement as to which of the

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[a] May, 1959, by Richard H. Bigelow.

[1] Ass't. Prof., North Carolina State College, Raleigh, N. C.

above solutions is most direct. A lighter section would be available in the 14-in series:

$$P_a \text{ (equivalent)} = 310 + (310 \times 6)\ 0.1214 = 536 \text{ kips}$$

Select 14 WF 142

To further compare Mr. Cunningham's method and the writer's method consider the problem below:

Select the lightest 12-in. column section for a buckling length of 30 ft with a 200-kip load acting on the Y-Y axis 6 in. from the X-X axis.

Professor Cunningham's method:

$$\text{Required A} = \frac{200 \times 1.5 + 200 \times 6 \times 0.2}{20} = 27 \text{ sq in.}$$

The 12 WF 92 having an area of 27.06 sq in. would be selected as the 1st trial.

$$\frac{L}{r} = \frac{30 \times 12}{3.08} = 116.9\ s_c = 10.37$$

$$\text{Required A} = \frac{200 \times \frac{20}{10.37} + 200 \times 6 \times .216}{20} = 32.25 \text{ sq in.}$$

The designer would probably try a 12 WF 120 with an area of 35.31 sq in. Repeating the calculation above for the 12 WF 120 he would find this section to suffice.

The author's method

$$P_a \text{ (equivalent)} = 200 + (200 \times 6)\ 0.1172 = 341 \text{ kips}$$

The next larger value for P_a in the 12 in. series is the 12 WF 120.

It should be noted that the 1st trial section as determined by Mr. Cunningham's method is three sections lighter than the final selection. Using the AISC bending factors directly would only place the first trial one section away from the final selection. One reason Mr. Cunningham's method may yield a first trial several sections from the final selected section is because his first trial calculation is independent of the buckling length. Again it will be left to the reader's judgement as to which of the above solutions is most direct. A lighter section would be available in the 14 in. series:

$$P_a \text{ (equivalent)} = 200 + (200 \times 6)\ 0.1164 = 340 \text{ K.}$$ Select 14 WF 95.

Reference is made to the solution in Mr. Chinn's discussion for a comparison with the solution below:

$$P_a \text{ (equivalent)} = 300 + (300 \times 4)\ 0.1354 = 462.5 \text{ Kips.}$$ Select 14 WF 111

For a full explanation of the writer's method the reader is referred to the original paper.

It has been the writer's experience that due to the infrequency of encountering bending about the Y-Y axis the compilation of a table for bending about this axis is not warranted. However, if a designer were interested in such a table he should calculate values of $\frac{P_a}{20\ S_{yy}}$ making sure to select the greatest value of $\frac{P_a}{20\ S_{yy}}$ in a particular series for a particular length.

ANALYSIS OF HAUNCHED OCTAGONAL GIRDER SPACE FRAMES[a]

Closure by Edward Y. W. Tsui

EDWARD Y. W. TSUI,[1] M. ASCE.—In answer to Mr. Pei's first question, it should be noted that all expressions for the torsional constant J_s (Eqs. 1 to 4) as well as moment of inertia I_s considered in this paper are assumed to be functions of the overall dimensions b and h_s of rectangular section only. These are consistent with the Assumption 1a, which states that the material is homogeneous, isotropic, etc. Accordingly, the gross area of the section was used to compute the stiffness factors (C_{ii}, A_{nn}), carry-over factors (C_{ij}, A_{mn}), etc.

In the case of reinforced concrete, whether or not the effect of reinforcement should be taken into consideration in calculating the influence coefficients is really a good question. Based on the writer's limited survey of related literature, very little work of this nature has been published. The writer is inclined to ignore the effect of steel in those terms involving C_{ij}, A_{mn}, $E I_s$ and $G J_s$ when they are used in solving the equilibrium equations in order to obtain the stress distribution. It is assumed that the percentage of reinforcement in each member of the structure does not change too greatly. If actual deformations (translations or rotations) are desired, however, more reasonable results can be obtained by using the composite I (= $I_{concrete}$ + I_{steel}) while the linear relationship between the flexural rigidity and torsional rigidity is still assumed to be a function of the overall dimensions of the section, namely

$$\frac{E\,I}{G\,J} = k \cong A\left[1+\left(\frac{h_s}{b}\right)^2\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

For example, if only the maximum vertical deflection of the girder due to static loads is desired, the deflection coefficients (λ_{13}, λ_{14}, λ_{15}) as shown in the Appendix are still valid, except that the total moment of inertia of concrete and steel should be used for I_c^x. It is apparent that not much extra work is involved in carrying out this procedure.

On the other hand, should the composite torsional constant be used simultaneously, it is expected that this would lead to complicated expressions for the influence coefficients which probably can not be evaluated analytically.

Mr. Pei is correct in his assumption that Tables I and II are limited to symmetrical haunched girders (as shown in Fig. 4).

The notation with respect to the symbol ρ has been missprinted and should be written as

$$\rho^x, \rho^y, \rho^z = \text{ratio } \Delta x/a, \Delta y/a, \Delta z/a, \text{ respectively}$$

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[a] June 1959 by E. Y. W. Tsui.

[1] Head, Applied Mechanics-Dynamics, Structures Dept., Lockheed Missiles and Space Div., Palo Alto, Calif.

BENDING MOMENTS ON SHELL BOUNDARIES[a]

Discussion by Gerard D. Galletly

GERARD D. GALLETLY,[1] M. ASCE.—The writer appreciates that the authors' main concern was not with symmetrically loaded shells of revolution. However, he believes that the readers would be interested in learning of some recent work done in this area.

The writer has computed, using an electronic digital computer, the stress and deformation (rotation and displacement) influence coefficients, over a wide range of geometric parameters, for the following rotational shells (Fig. 1):

1. Toroidal shells. Both positive- and negative-Gaussian curvature shells have been treated. The edge deformation quantities resulting from this work have been published.[2,3] A set of the interior stress and deformation influence coefficients, which were too voluminous to publish in a paper, were denoted to the ASCE library.
2. Truncated ellipsoidal shells. The edge deformation quantities at the large and small ends of these shells may be found.[4] The writer has also obtained the relevant interior influence coefficients for these shells but has not published them as yet.
3. Truncated spherical shells. Both the stress and deformation influence coefficients, at and near both ends of truncated spherical shells, are available.[5]

It is to be noted that the loadings considered for the three shells were uniformly distributed edge moments and shears and uniform internal pressure.

The writer would also like to mention that, as a result of Taylor's work,[6] the stress analysis of both complete and truncated cones is now a relatively easy matter.

The utility of the above influence coefficients, together with some applications, has recently been described.[7] A summary of some recent asymptotic integration solutions, together with a comparison of results obtained on a digital computer, may be found.[8]

It is also to be noted that the use of the membrane solution as a particular integral of the differential equations of equilibrium does not always yield good

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[a] October, 1959, by H. H. Bleich and M. G. Salvadori.
[1] Galletly, G. D. Engr., Shell Development Co., Emeryville, Calif.
[2] Galletly, G. D., ASME Paper No. 59-Pet-2.
[3] Galletly, G. D., ASME Paper No. 59-Pet-3.
[4] Galletly, G. D., Welding Research Council Bulletin No. 54, October, 1959.
[5] Galletly, G. D. ASME Paper No. 59-SA-4.
[6] Taylor, C. E., "Simplification of the Analysis of Stress in Conical Shells" University of Illinois, Urbana, Illinois, 1958, unpublished.
[7] Galletly, G. D., ASME Paper No. 59-A-163.

results. Two instances of this, for toroidal shells subjected to internal pressure, are given.[8,9]

In section 3 of their paper, the authors give some approximate rules for judging when the cylinder or Geckeler approximation should give reasonable results when it is applied to rotational shells. The writer would like to comment on these rules.

First, he does not understand their limitation of "ϕ_0 not greater than 20°." Reference to p. 467 of Timoshenko's "Theory of Plates and Shells" shows that ϕ_0 should be large rather than small. This also agrees with the Geckeler-

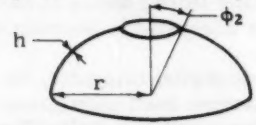
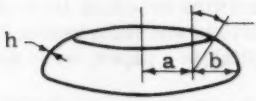
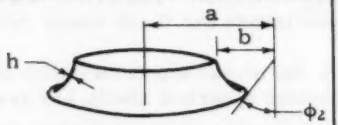
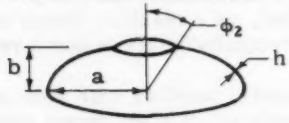
Type of Shell	Range of Parameters
Open-Crown Hemisphere 	ϕ_2 = 50°, 40°, 30°, 20°, 10° 15 < r/h < 250
Positive-Curvature Toroid 	ϕ_2 = 75°, 60°, 45°, 30°, 15° a/b = 4, 5, 6, 7, 8 5 < b/h < 50
Negative-Curvature Toroid 	ϕ_2 = 75°, 60°, 45°, 30°, 15° a/b = 3, 4, 5, 6, 7 5 < b/h < 50
Open-Crown Ellipsoid 	ϕ_2 = 50°, 40°, 30°, 20°, 10° a/b = 2, 3 15 < a/h < 250

FIG. 1

type solution obtained by Horvay and his associates.[10] The latter state that ϕ_0 should lie between $\pi/4$ and $3\pi/4$ for their method to give good results. For small ϕ_0 = angles, one can use Hetenyi's method (see p. 473 of Timoshenko's "Theory of Plates and Shells") or, better still, the solutions in terms of Bessel functions.[8]

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[8] Galletly, G. D. and Radok, J.R.M., ASME Paper No. 59-APM-30.

[9] Galletly, G. D., Trans. ASME, Vol. 81, Series B. No. 1, February, 1959, p. 51.

[10] Horvay G., Linkous C. and Born, J. S., Journal of Applied Mechanics, March 1956, p. 68.

Assuming the authors' formula for the penetration p to be valid, the writer would now like to illustrate that item c) of their section 3 (that is, "shell hugging tangent sphere in the boundary region") does not necessarily hold. Two shell-types will be considered: (a) an ellipsoid of revolution (their Fig. 8a) and (b) a toroidal shell of negative Gaussian curvature (their Fig. 8b).

For the ellipsoidal shell, it will be assumed that a/h = 50. Then, at $\phi = 90°$, $p = 1.79\sqrt{h/R_2} < 1/3$. One finds,[4] at $\phi = 90°$;

$$\left.\begin{array}{l} \text{2:1 ellipsoid } E\,h^2\,\theta_m = 60.6 \\ \text{3:1 ellipsoid } E\,h^2\,\theta_m = 63.7 \end{array}\right\} \text{ with } \nu = 0.3 \text{ and } M_o = 1$$

The above values are actually for truncated ellipsoids with semi-opening angles of 10°. However, the values will be practically identical with those for complete ellipsoids, as the edge effect is localized.

Using now the cylinder approximation, one has $E\,h^2\,\theta_m = 9.1\,M_o\sqrt{R_2/h} = 64.3$ at $\phi = 90°$ and with $\nu = 0$. The agreement between the approximate and the accurate values of θ_m is thus quite good, which is contrary to the authors' prediction.

Considering the toroidal shell next, two cases with a/h = 60 will be treated. For these, $p < 1/3$ also. The geometric characteristics of the two shells are: (i) $\frac{R_2}{R_1} = 6$, $\frac{R_1}{h} = 10$ and (ii) $\frac{R_2}{R_1} = 2$, $\frac{R_1}{h} = 30$. (The notation is that used by the authors.) One additional parameter will be included. This is the angle ϕ_2, which is as shown in Fig. 1 of this discussion.

One has[3] (for $M_o = 1$, $\nu = 0.3$, $E\,h^2\,\theta_m$

ϕ_2	Case (i)	Case (ii)
15°	77.5	66.1
30°	92.2	66.2
45°	140.1	66.3

Employing the cylinder approximation, one has (for $M_o = 1$, $\nu = 0$),

$$E\,h^2\,\theta_m = 9.1\,\sqrt{60} = 70.4$$

Thus for case (ii), the cylinder approximation gives good results, irrespective of the angle ϕ_2. This is not the case for case (i), however. Reference to the complete set of influence coefficients mentioned earlier (donated to the ASCE library) shows that the moment does not change sign for the case (i) shell, while the penetration p for the case (ii) shell is about 27°. This explains why the cylinder approximation is not very good for the case (i) shell and shows that the moment penetration is not simply a function of $\sqrt{h/R_2}$ in the general case. For small values of R_2/R_1, however, (2 in case (ii) above) the authors' formula for the moment penetration seems reasonable.

With regard to hyperbolic paraboloidal shells, the writer would like to mention an additional reference on approximate estimates of edge bending effects.

This is the work of C. G. J. Vreedenburgh.[11] The latter author approximates the hyperbolic paraboloid in a small region by a translation surface with curvatures k_1 and k_2. He then derives approximate formulae for the edge effects. He states that the usefulness of the formulas has been demonstrated experimentally in his laboratory at Delft.

11 Vreedenburgh, C.G.J., Philips Technical Review, Vol. 20, No. 1, 1958/59, pp 9-17.

CUT-OFF POINTS FOR COVER PLATES[a]

Corrections

CORRECTIONS.—On page 24, please change item 2 to the following:

2. Determine $a = \frac{f_a}{f_s}$

f_a = allowable stresses at cut-off point

= 13.5 kips per square inch (Spec.)

$a = \frac{13.5}{29.5} = .458$

[a] November, 1959, by Nat T. Kanarek.

PROCEEDINGS PAPERS

The technical papers published in the past year are identified by number below. Technical-division sponsorship is indicated by an abbreviation at the end of each Paper Number, the symbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Pipeline (PL), Power (PO), Sanitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways and Harbors (WW), divisions. Papers sponsored by the Department of Conditions of Practice are identified by the symbols (PP). For titles and order coupons, refer to the appropriate issue of "Civil Engineering." Beginning with Volume 82 (January 1956) papers were published in Journals of the various Technical Divisions. To locate papers in the Journals, the symbols after the paper number are followed by a numeral designating the issue of a particular Journal in which the paper appeared. For example, Paper 2270 is identified as 2270(ST9) which indicates that the paper is contained in the ninth issue of the Journal of the Structural Division during 1959.

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MARCH: 1960(HY3), 1961(HY3), 1962(HY3), 1963(IR1), 1964(IR1), 1965(IR1), 1966(IR1), 1967(SA2), 1968(SA2), 1969(ST3), 1970(ST3), 1971(ST3), 1972(ST3), 1973(ST3), 1974(ST3), 1975(ST3), 1976(WW1), 1977(WW1), 1978(WW1), 1979(WW1), 1980(WW1), 1981(WW1), 1982(WW1), 1983(WW1), 1984(SA2), 1985(SA2)[c], 1986 (IR1)[c], 1987(WW1)[c], 1988(ST3)[c], 1989(HY3)[c].

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MAY: 2014(AT2), 2015(AT2), 2016(AT2), 2017(HY5), 2018(HY5), 2019(HY5), 2020(HY5), 2021(HY5), 2022(HY5), 2023(PL2), 2024(PL2), 2025(PL2), 2026(PP1), 2027(PP1), 2028(PP1), 2029(PP1), 2030(SA3), 2031(SA3), 2032(SA3), 2033(SA3), 2034(ST5), 2035(ST5), 2036(ST5), 2037(ST5), 2038(PL2), 2039(PL2), 2040(AT2)[c]. 2041(PL2)[c], 2042(PP1)[c], 2043(ST5)[c], 2044(SA3)[c], 2045(HY5)[c], 2046(PP1), 2047(PP1).

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JULY: 2079(HY7), 2080(HY7), 2081(HY7), 2082(HY7), 2083(HY7), 2084(HY7), 2085(HY7), 2086(SA4), 2087 (SA4), 2088(SA4), 2089(SA4), 2090(SA4), 2091(EM3), 2092(EM3), 2093(EM3), 2094(EM3), 2095(EM3), 2096 (EM3), 2097(HY7)[c], 2098(SA4)[c], 2099(EM3)[c], 2100(AT3), 2101(AT3), 2102(AT3), 2103(AT3), 2104(AT3), 2105(AT3), 2106(AT3), 2107(AT3), 2108(AT3), 2109(AT3), 2110(AT3), 2111(AT3), 2112(AT3),2113(AT3), 2114(AT3), 2115(AT3), 2116(AT3), 2117(AT3), 2118(AT3), 2119(AT3), 2120(AT3), 2121(AT3),2122(AT3), 2123(AT3), 2124(AT3), 2125(AT3).

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OCTOBER: 2189(AT4), 2190(AT4), 2191(AT4), 2192(AT4), 2193(AT4), 2194(EM4), 2195(EM4), 2196(EM4), 2197(EM4), 2198(EM4), 2199(EM4), 2200(HY10), 2201(HY10), 2202(HY10), 2203(PL3), 2204(PL3), 2205 (PL3), 2206(PO5), 2207(PO5), 2208(PO5), 2209(PO5), 2210(SM5), 2211(SM5), 2212(SM5), 2213(SM5), 2214 (SM5), 2215(SM5), 2216(SM5), 2217(SM5), 2218(ST8), 2219(ST8), 2220(EM4), 2221(ST8), 2222(ST8), 2223 (ST8), 2224(HY10), 2225(HY10), 2226(PO5), 2227(PO5), 2228(PO5), 2229(ST8), 2230(EM4), 2231(EM4), 2232(AT4)[c], 2233(PL3)[c], 2234(EM4)[c], 2235(HY10)[c], 2236(SM5)[c], 2237(ST8)[c], 2238(PO6)[c], 2239(ST8), 2240 (PL3).

NOVEMBER: 2241(HY11), 2242(HY11), 2243(HY11), 2244(HY11), 2245(HY11), 2246(SA6), 2247(SA6), 2248 (SA6), 2249(SA6), 2250(SA6), 2251(SA6), 2252(SA6), 2253(SA6), 2254(SA6), 2255(SA6), 2256(ST9), 2257(ST9), 2258(ST9), 2259(ST9), 2260(HY11), 2261(ST9)[c], 2262(ST9), 2263(HY11), 2264(ST9), 2265(HY11), 2266(SA6), 2267(SA6), 2268(SA6), 2269(HY11)[c], 2270(ST9).

DECEMBER: 2271(HY12)[c], 2272(CP2), 2273(HW4), 2274(HW4), 2275(HW4), 2276(HW4), 2277(HW4), 2278 (HW4), 2279(HW4), 2280(HW4), 2281(IR4), 2282(IR4), 2283(IR4), 2284(IR4), 2285(PO6), 2286(PO6), 2287 (PO6), 2288(PO6), 2289(PO6), 2290(PO6), 2291(PO6), 2292(SM6), 2293(SM6), 2294(SM6), 2295(SM6), 2296 (SM6), 2297(WW4), 2298(WW4), 2299(WW4), 2300(WW4), 2301(WW4), 2302(WW4), 2303(WW4), 2304(HW4), 2305(ST10), 2306(CP2), 2307(CP2), 2308(ST10), 2309(CP2), 2310(HY12), 2311(HY12), 2312(PO6), 2313(PO6), 2314(ST10), 2315(HY12), 2316(HY12), 2317(HY12), 2318(WW4), 2319(SM6), 2320(SM6), 2321(ST10), 2322 (ST10), 2323(HW4)[c], 2324(CP2)[c], 2325(SM6)[c], 2326(WW4)[c], 2327(IR4)[c], 2328(PO6)[c], 2329(ST10)[c], 2330 (CP2).

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FEBRUARY: 2355(CO1), 2356(CO1), 2357(CO1), 2358(CO1), 2359(CO1), 2360(CO1), 2361(PO1), 2362(HY2), 2363(ST2), 2364(HY2), 2365(SU1), 2366(HY2), 2367(SU1), 2368(SM1), 2369(HY2), 2370(SU1), 2371(HY2), 2372(PO1), 2373(SM1), 2374(HY2), 2375(PO1), 2376(HY2), 2377(CO1)[c], 2378(SU1), 2379(SU1), 2380(SU1), 2381(HY2)[c], 2382(ST2), 2383(SU1), 2384(ST2), 2385(SU1)[c], 2386(SU1), 2387(SU1), 2388(SU1), 2389(SM1), 2390(ST2)[c], 2391(SM1)[c], 2392(PO1)[c].

c. Discussion of several papers, grouped by divisions.

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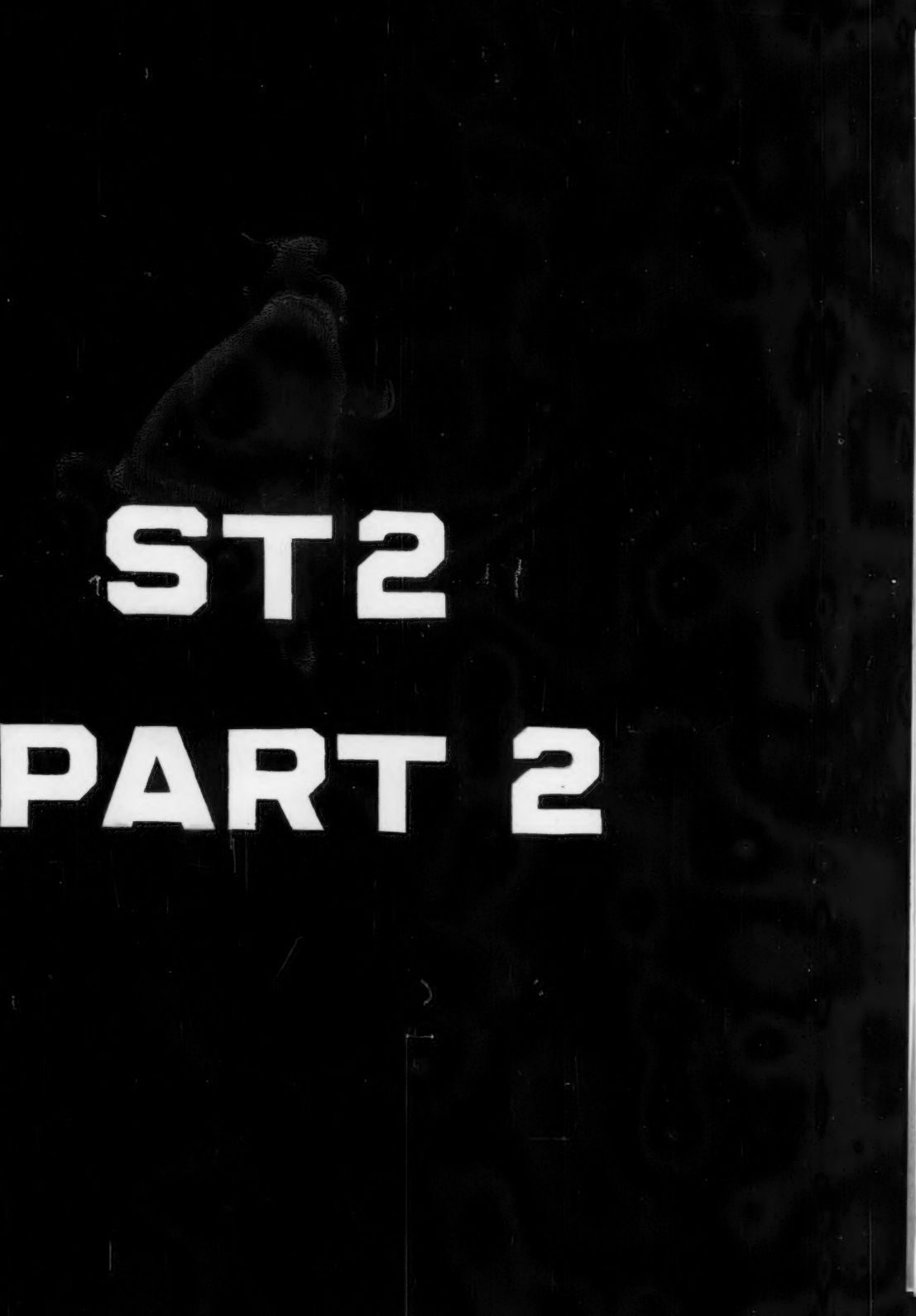
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Your attention is invited

NEWS OF THE STRUCTURAL DIVISION OF ASCE

JOURNAL OF THE STRUCTURAL DIVISION
PROCEEDINGS OF THE AMERICAN SOCIETY OF CIVIL ENGINEERS

DIVISION ACTIVITIES
STRUCTURAL DIVISION
Proceedings of the American Society of Civil Engineers

NEWS

February, 1960

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ASCE CONVENTION, NEW ORLEANS, MARCH 6-11, 1960

The Structural Division program for the New Orleans ASCE Convention will have as its theme the subject of Prestressed Concrete" and will be co-sponsored by the American Concrete Institute and the Prestressed Concrete Institute.

B. M. Dornblatt, Structural Division Session Program Chairman, and those who assisted him have prepared an excellent program which deserves our support. The program will cover the field of Experimental Research, Buildings and Structures, Bridges, Construction and recent developments in Prestressed Concrete. The complete program for the Convention appears in CIVIL ENGINEERING.

In addition to the main theme, two joint sessions have been arranged. The first with the Construction Division on Wednesday afternoon, covering both prestressed concrete and other unusual bridge construction projects, such as the Champlain Bridge, Montreal, the Oneida Lake Bridge, and the construction, design and economics of cast-in-place post tensioned concrete bridges of over 100-ft. span.

On Thursday afternoon, the second joint session will be held. This will be with the Power Division, and consists of a series of papers on tower design and construction.

The Structural Division program starts with the morning session on Tuesday, March 8, and concludes on Friday morning, March 11. During this period four

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sessions are devoted exclusively to some twelve papers on the subject of Prestressed Concrete. Such topics are discussed as the use of prestressed concrete in dam construction, railroad ties, girders, cylindrical shell roof structures, piling and bridges. The question of long time creep, shear tests on beams with web reinforcing, fire resistance and the laboratory test of a two-span continuous precast prestressed bridge are investigated.

Among the authors participating in the program are Philip M. Grennan, John B. Skilling, T. Y. Lin and W. J. Talbot, Dr. G. Hernandez, Hubert Woods, Alan Mattock, W. E. (Bill) Dean, R. M. Dubois, Eric Molke, Morris Shupack, Wayne Henneberger, Dave Henderson, Al Parme, O. C. Zienkiewicz and G. M. Magee. With this array of talent an interesting, informative and lively meeting is anticipated.

EXECUTIVE COMMITTEE

The Executive Committee of the Structural Division for the year beginning October 1959 and ending October 1960 consists of the following members: Robert D. Dewell, Chairman; Emerson J. Ruble, Vice-Chairman; George S. Vincent; Nathan D. Whitman, Jr. and Charles T. G. Looney, Secretary. Elmer K. Timby, last year's Vice-Chairman of the Executive Committee, has been elected to the Board of Direction, ASCE. He has been appointed contact member of the Board of Direction to the Structural Division.

A three-day meeting of the Executive Committee was held in Washington, D. C. during the October ASCE Convention. Some of the highlights of that meeting were as follows:

The work of the Task Committee on Bridge Loadings was reviewed. It was decided that this task committee proceed with a review of existing design specifications with a view of simplifying the bridge loadings now in use.

The Executive Committee recommended the formation of a joint committee, consisting of the three Administrative Chairmen of the Committees on Wood, Metals and Masonry, to study and make recommendations concerning the use of chemical resins as bonding agents for structural joints.

The work of the Task Committee on Factors of Safety was discussed. This task committee has been active for a number of years and is encountering difficulty in writing a final report. There is considerable difference in viewpoints between members of this committee. It was felt that a study of this subject should be continued and related to the field of ultimate strength design methods.

Of interest to committee chairmen is the new policy as regards approval of new committee appointments. The chairman of the Administrative Committee notifies the contact member for the Executive Committee of the proposed new member and he in turn notifies the other members of the Executive Committee. If the contact member receives no word from the members of the Executive Committee within ten days, it is assumed that the new member is approved and the contact member so notifies the Chairman of the Administrative Committee.

Frank Baron discussed the desirability of establishing permanent working committees of the IABSE (International Association for Bridge and Structural Engineering) in Europe to cooperate with parallel committees of the Structural Division of the ASCE. The Executive Committee agreed on the desirability of this arrangement. The IABSE will appoint a member of the American branch to look into this matter.

There has been some criticism of the long delay of the Committee on Publications in reviewing papers. The records show that there are some 80 papers in the process of review. The Executive Committee is studying this complex problem with the hope that a simpler and faster means of getting a paper reviewed will result.

The Executive Committee also heard reports on the activities of the following committees: Masonry and Reinforced Concrete, Loads and Stresses, Metals, Buildings and Nuclear Structures. A considerable amount of time was devoted to a discussion of the Structural Division programs for this year's ASCE conventions. The location, date, theme and sessions program chairman for these meetings are as follows:

New Orleans	- March 6-11	- Prestressed Concrete	- B. M. Dornblatt
Reno	- June 19-24	- Loads and Stresses	- Stewart Mitchell
Boston	- Oct. 9-14	- Basic Research	- Saul Namyet

COMMITTEE ON ELECTRONIC COMPUTATION

The Committee on Electronic Computation of the Structural Division is very pleased with the enthusiastic response to the invitation for papers to be presented at the Second Structural Division Conference on Electronic Computation. As previously announced, the conference is to be held on September 8 and 9, 1960 at the Pittsburgh Hilton Hotel, and the Pittsburgh Section of ASCE will serve as host.

Approximately seventy-five abstracts have been received, and these cover the fields of structural design and analysis; mathematical methods; and problems of organizations, education, program exchange, etc. It is anticipated that twenty-five to thirty papers will be selected for the conference. A review committee has been formed and has already met to go over the abstracts. Final selections will not be made until after the submission of the complete manuscripts which are due April 15.

The Pittsburgh Section is progressing very effectively in planning for their functions as host. They have set up the following committees: Hotel Arrangements, Registration and Information, Entertainment, Finance, Publicity, Reception, and Student Activities. The complete facilities of the ballrooms of the Pittsburgh Hilton have been reserved. Because of the size of floor space available, all technical sessions as well as luncheons will be held on the same floor.

The Committee on Electronic Computation is looking forward to a successful conference, and is anticipating an attendance of over five hundred engineers.

COLUMN RESEARCH COUNCIL

The Column Research Council held a meeting at the Board Room of the AISC in New York City on December 10, 1959. The purpose of the meeting was to make final editorial changes and to arrange for the printing of the "Guide to Design Criteria for Metal Compression Members." This Guide will be of special interest to structural engineers. Details of how to obtain this guide will appear in a future issue of the Newsletter. The following excerpt is from the foreword to the Guide:

"The results of Column Research Council projects taken together with the results of many other investigations on the buckling strength of compression elements brought about a situation in which there is a vast amount of research information that is not applied in practice by the designer of columns or compression elements. Column Research Council has as its goal the development of practical design procedures that are as simple as possible and yet consistent with accurate and safe predictions of structural strength. In keeping with this goal there came into being the idea of preparing this general 'guide', the initial outlines of which were formulated in 1956."

"An adopted policy of Column Research Council has been the avoidance of any attempt to supplant existing specifications. The goal is to serve all specifications by offering guidance toward improvement by revision. An effort is made in the Guide to present both 'complex' and 'simple' procedures of design calculation and to assess the limitations of the latter. It is left to the specification writers to choose the procedures that are deemed preferable."

STRUCTURAL DIVISION JOURNAL

The next issue of the Structural Division Newsletter will be published in May. Material for this issue must reach the editor before the end of March. Committee Chairmen are requested to send to the editor news items and minutes of any task committee meetings.

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